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On the rotation-two-component Camassa–Holm system modelling the equatorial water waves

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ABSTRACT

In this paper, a modified two-component Camassa–Holm system with the effect of the Coriolis force in the rotating fluid is derived, which is a model in the equatorial water waves. The effects of the Coriolis force caused by the Earth's rotation and nonlocal nonlinearities on blow-up criteria and wave-breaking phenomena are then investigated. Our refined analysis relies on the method of characteristics and conserved quantities and is proceeded with the Riccati-type differential inequality. Finally, conditions which guarantee the permanent waves are obtained by using a method of the Lyapunov function.

1. Introduction

It is known that for the geophysical water waves the forces with primary influence are the gravity and the Coriolis force induced by the Earth's rotation. When considering

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waves propagating in the equatorial ocean regions throughout the extent of the Pacific Ocean, it is found however that the Equatorial Undercurrent (EUC) [14,28,36] is one essential feature and the effect of the Coriolis force is small, because of the smallness of the variation in latitude of the EUC in the equatorial region. In this case, one can use the approximation by the f-plane governing equations rather than the β-plane approximation to capture the peculiarity of the EUC flow [8]. There have recently appeared several works involving steady periodic rotational Equatorial water waves in the f-plane on topics like existence [9,25,32], regularity of free surface and of the stream lines [33], symmetry [26] and stability [20], for example.

It is our purpose in the present paper to derive the following rotation-two-component Camassa–Holm (R2CH) system

$$\begin{cases} u_t - u_{xxt} - Au_x + 3uu_x = \sigma(2u_x u_{xx} + uu_{xxx}) - \mu u_{xxx} - (1-2\Omega A)\rho\rho_x + 2\Omega\rho(\rho u)_x, \\ \rho_t + (\rho u)_x = 0 \end{cases} \tag{1.1}$$

from the f-plane governing equations for the geophysical water waves which admit a constant underlying current and then investigate the properties of breaking and the permanent waves for the solutions of the established model. And here the function $u(t,x)$ is the fluid velocity in the x-direction, $\rho(t,x)$ is related to the free surface elevation from equilibrium, the parameter A characterizes a linear underlying shear flow, the real dimensionless constant σ is a parameter which provides the competition, or balance, in fluid convection between nonlinear steepening and amplification due to stretching, μ is a nondimensional parameter and Ω characterizes the constant rotational speed of the Earth. The boundary assumptions associated with (1.1) are $u \to 0$, $\rho \to 1$ as $|x| \to \infty$. There are at least three conservation laws associated with the system in (1.1):

$$E(u,\rho) = \frac{1}{2}\int_{\mathbb{R}} \left(u^2 + u_x^2 + (1-2\Omega A)(\rho-1)^2\right) dx, \tag{1.2}$$

$$I_1(u,\rho) = \int_{\mathbb{R}} \left(u - \Omega(\rho-1)^2\right) dx, \qquad \text{and} \tag{1.3}$$

$$I_2(u,\rho) = \int_{\mathbb{R}} (\rho-1)dx. \tag{1.4}$$

When $\Omega = 0$, i.e. without considering effect of the Earth's rotation, then the following functional is conserved.

$$F(u,\rho) = \frac{1}{2}\int_{\mathbb{R}} \left(u^3 + \sigma u u_x^2 - Au^2 - \mu u_x^2 + 2(\rho-1)u + u(\rho-1)^2\right) dx. \tag{1.5}$$