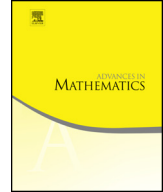




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# Convexity estimates for mean curvature flow with free boundary



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## ABSTRACT

We prove the estimates of [3] and [2] for finite-time singularities of mean-convex, mean curvature flow with free boundary in a barrier  $S$ . Here  $S$  can be any embedded, oriented surface in  $R^{n+1}$  of bounded geometry and positive inscribed radius. We also prove the estimate [4] in the case of convex flows and  $S = S^n$ , which gives an alternative proof to [6].

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## 1. Introduction

We are interested in immersed, mean-convex, mean curvature flow with free boundary in a surface  $S$ . We reprove the estimates in [3], and [2] for this class of flows. These provide very direct, general pinching results for limit flows at singularities, and require no embeddedness or curvature assumptions. We further prove the estimates in [4] when  $S$  is the sphere.

Consider a smooth, embedded, oriented hypersurface  $S \subset R^{n+1}$ , with choice of normal  $\nu_S$ , having bounded geometry and positive inscribed radius. We refer to  $S$  as the

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*barrier surface*. If  $\Sigma^n \subset R^{n+1}$  is a compact, mean-convex hypersurface with boundary, we say  $\Sigma$  *meets  $S$  orthogonally* if  $\partial\Sigma \subset S$ , and the outer normal of  $\partial\Sigma \subset \Sigma$  coincides with  $\nu_S$ .

Let  $\Sigma_0 = \Sigma$  meet the barrier  $S$  orthogonally. Then the mean curvature flow of  $\Sigma_0$ , with free-boundary in  $S$ , is a family of immersions  $F_t : \Sigma_0 \times [0, T) \rightarrow R^{n+1}$  such that

$$\begin{aligned}\partial_t F_t &= -H\nu, \text{ for all } p \in \Sigma, t > 0 \\ F_t(\Sigma) &\text{ meets } S \text{ orthogonally for all } t \geq 0 \\ F_0 &\equiv \text{Id}_{\Sigma_0}.\end{aligned}$$

Here  $H$  is the mean curvature, and  $\nu$  the outer normal, oriented so that  $\mathbf{H} = -H\nu$  is the mean curvature vector. We often write  $\Sigma_t = F_t(\Sigma)$ , and will identify the surface with its immersion.

It was shown by Stahl [7] that the mean curvature flow with free-boundary in  $S$  always exists on some maximal time interval  $[0, T)$ , for  $T \leq \infty$ , such that if  $T < \infty$  then necessarily  $\max_{\Sigma_t} |A| \rightarrow \infty$  as  $t \rightarrow T$ . Here  $|A|$  is the norm of the second fundamental form  $A$ .

Type-I tangent flows of mean curvature flow with free boundary have been classified by Buckland [1]. Our convexity estimates work towards classifying type-II limit flows with free boundary. Stahl [6] has shown Theorem 1.6 using a different method.

We prove the following theorems concerning the mean curvature flow of  $\Sigma_0$  with free-boundary in  $S$ . Throughout the duration of this paper we assume  $\Sigma_0$  is compact, mean-convex.

**Theorem 1.1.** *There are constants  $\alpha = \alpha(S) \geq 0$  and  $C = C(S, \Sigma_0)$  so that*

$$\max_{\Sigma_t} \frac{|A|}{H} \leq Ce^{\alpha t} \tag{1}$$

*for all time of existence. In particular, if  $T < \infty$ , then*

$$|A| \leq C(S, \Sigma_0, T)H.$$

**Definition 1.1.1.** Given a vector  $\mu \in R^n$ , and  $k \in \{1, \dots, n\}$ , we let

$$s_k(\mu) = \sum_{1 \leq i_1 < \dots < i_k \leq n} \mu_{i_1} \cdots \mu_{i_k}$$

be the  $k$ -th symmetric polynomial of  $\mu$ . We adopt the convention that  $s_0 \equiv 1$ . If  $s_{k-1}(\mu) \neq 0$ , we let

$$q_k(\mu) = \frac{s_k(\mu)}{s_{k-1}(\mu)}.$$

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