Advances in Mathematics 294 (2016) 1–36

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Convexity estimates for mean curvature flow with free boundary

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A R T I C L E I N F O

Article history:
Received 15 November 2014
Received in revised form 10 February 2016
Accepted 23 February 2016
Available online 8 March 2016
Communicated by Camillo De Lellis

Keywords:
Mean curvature flow
Convexity estimates
Free boundary
Limit flows

A B S T R A C T

We prove the estimates of [3] and [2] for finite-time singularities of mean-convex, mean curvature flow with free boundary in a barrier S. Here S can be any embedded, oriented surface in R^{n+1} of bounded geometry and positive inscribed radius. We also prove the estimate [4] in the case of convex flows and $S = S^n$, which gives an alternative proof to [6].

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1. Introduction

We are interested in immersed, mean-convex, mean curvature flow with free boundary in a surface S. We reprove the estimates in [3], and [2] for this class of flows. These provide very direct, general pinching results for limit flows at singularities, and require no embeddedness or curvature assumptions. We further prove the estimates in [4] when S is the sphere.

Consider a smooth, embedded, oriented hypersurface $S \subset R^{n+1}$, with choice of normal ν_S, having bounded geometry and positive inscribed radius. We refer to S as the

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http://dx.doi.org/10.1016/j.aim.2016.02.026
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barrier surface. If $\Sigma^n \subset R^{n+1}$ is a compact, mean-convex hypersurface with boundary, we say Σ *meets S orthogonally* if $\partial\Sigma \subset S$, and the outer normal of $\partial\Sigma \subset \Sigma$ coincides with ν_S.

Let $\Sigma_0 = \Sigma$ meet the barrier S orthogonally. Then the mean curvature flow of Σ_0, with free-boundary in S, is a family of immersions $F_t : \Sigma_0 \times [0, T) \to R^{n+1}$ such that

$$\partial_t F_t = -H\nu, \text{ for all } p \in \Sigma, t > 0$$

$$F_t(\Sigma) \text{ meets } S \text{ orthogonally for all } t \geq 0$$

$$F_0 \equiv \mathrm{Id}_{\Sigma_0}.$$

Here H is the mean curvature, and ν the outer normal, oriented so that $\mathbf{H} = -H\nu$ is the mean curvature vector. We often write $\Sigma_t = F_t(\Sigma)$, and will identify the surface with its immersion.

It was shown by Stahl [7] that the mean curvature flow with free-boundary in S always exists on some maximal time interval $[0, T)$, for $T \leq \infty$, such that if $T < \infty$ then necessarily $\max_{\Sigma_t} |A| \to \infty$ as $t \to T$. Here $|A|$ is the norm of the second fundamental form A.

Type-I tangent flows of mean curvature flow with free boundary have been classified by Buckland [1]. Our convexity estimates work towards classifying type-II limit flows with free boundary. Stahl [6] has shown Theorem 1.6 using a different method.

We prove the following theorems concerning the mean curvature flow of Σ_0 with free-boundary in S. Throughout the duration of this paper we assume Σ_0 is compact, mean-convex.

Theorem 1.1. *There are constants $\alpha = \alpha(S) \geq 0$ and $C = C(S, \Sigma_0)$ so that*

$$\max_{\Sigma_t} \frac{|A|}{H} \leq Ce^{\alpha t} \tag{1}$$

for all time of existence. In particular, if $T < \infty$, then

$$|A| \leq C(S, \Sigma_0, T)H.$$

Definition 1.1.1. Given a vector $\mu \in R^n$, and $k \in \{1, \ldots, n\}$, we let

$$s_k(\mu) = \sum_{1 \leq i_1 < \ldots < i_k \leq n} \mu_{i_1} \cdots \mu_{i_k}$$

be the k-th symmetric polynomial of μ. We adopt the convention that $s_0 \equiv 1$. If $s_{k-1}(\mu) \neq 0$, we let

$$q_k(\mu) = \frac{s_k(\mu)}{s_{k-1}(\mu)}.$$

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