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Rigidity of self-shrinkers and translating solitons of mean curvature flows

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ABSTRACT

In this paper, we prove that any complete m-dimensional spacelike self-shrinkers in pseudo-Euclidean spaces $\mathbb{R}^{m+n}_n$ must be affine planes, and there exists no complete m-dimensional spacelike translating soliton in $\mathbb{R}^{m+n}_n$. These results are proved by using a new Omori–Yau maximal principle. We also derive a rigidity theorem of self-shrinking hypersurfaces in Euclidean space with Gauss image lies in a regular ball.

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1. Introduction

The mean curvature flow (MCF) in Euclidean space (pseudo-Euclidean space resp.) is a one-parameter family of immersions $X_t = X(\cdot, t) : M^m \to \mathbb{R}^{m+n}$ ($\mathbb{R}^{m+n}_n$ resp.) with the corresponding image $M_t = X_t(M)$ such that

$$\begin{cases} \dfrac{d}{dt}X(x,t) = H(x,t) \quad x \in M, \\ X(x,0) = X(x), \end{cases} \tag{1.1}$$

is satisfied, here $H(x,t)$ is the mean curvature vector of M_t at $X(x,t)$ in $\mathbb{R}^{m+n}$ ($\mathbb{R}^{m+n}_n$ resp.).

M^m is said to be a self-shrinker in $\mathbb{R}^{m+n}$ (spacelike self-shrinker in $\mathbb{R}^{m+n}_n$ resp.) if it satisfies a quasi-linear elliptic system

$$H = -X^N, \tag{1.2}$$

which is an important class of solutions to (1.1), where X^N is the normal part of X.

We call M^m a translating soliton in $\mathbb{R}^{m+n}$ if it satisfies

$$H = -v^N, \tag{1.3}$$

where H is the mean curvature vector of M and v is a fixed vector in $\mathbb{R}^{m+n}$ with unit length and v^N denotes the orthogonal projection of v onto the normal bundle of M.

Self-similar solutions to the MCF play an important role in understanding the behavior of the flow since they often occur as singularities. The subject of self-shrinkers in Euclidean spaces are also closely related with the theory of minimal submanifolds (see e.g. [4,16]). There is a plenty of works on the classification and uniqueness problem for self-shrinkers and translating solitons in Euclidean spaces (see e.g. [1,22,30,28,23,16,9,27,8,19,20,31,13]).

On the other hand, there are many works on the rigidity problem for complete spacelike submanifolds. Calabi [7] proposed the rigidity problem for complete spacelike extremal hypersurfaces in Minkowski space $\mathbb{R}^{m+1}_1$. He proved that such hypersurfaces have to be hyperplanes when $m \leq 4$. Cheng–Yau [15] solved the problem for all m, in sharp contrast to the situation of Euclidean space. Later, Jost–Xin [24] generalized the results to higher codimensions.

In view of the above two aspects, it is then natural to study the corresponding rigidity problems for spacelike self-shrinkers. Chau–Chen–Yuan [9] and Huang–Wang [21] proved that any spacelike entire graphic Lagrangian self-shrinkers must be flat under the condition that the Hessian of the potential function is bounded below quadratically by different methods. Ding–Wang in [18] derived rigidity results for spacelike entire graphs under subexponential decay condition. Ding–Xin [19] showed that such Lagrangian self-shrinkers are flat by removing the additional condition in [9] and [21]. Some rigidity and