

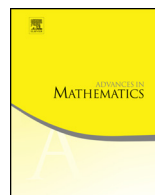


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Rigidity of self-shrinkers and translating solitons of mean curvature flows



Qun Chen^{a,*}, Hongbing Qiu^{a,b}

^a School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China

^b Max Planck Institute for Mathematics in the Sciences, Inselstr. 22, D-04103 Leipzig, Germany

ARTICLE INFO

Article history:

Received 19 April 2015

Received in revised form 4 March 2016

Accepted 4 March 2016

Available online 16 March 2016

Communicated by Gang Tian

MSC:

53C44

53C40

53C43

Keywords:

Self-shrinker

Translating soliton

Rigidity

Omori–Yau maximum principle

V-harmonic map

ABSTRACT

In this paper, we prove that any complete m -dimensional spacelike self-shrinkers in pseudo-Euclidean spaces \mathbb{R}_n^{m+n} must be affine planes, and there exists no complete m -dimensional spacelike translating soliton in \mathbb{R}_n^{m+n} . These results are proved by using a new Omori–Yau maximal principle. We also derive a rigidity theorem of self-shrinking hypersurfaces in Euclidean space with Gauss image lies in a regular ball.

© 2016 Elsevier Inc. All rights reserved.

* Corresponding author.

E-mail addresses: qunchen@whu.edu.cn (Q. Chen), hbqiu@whu.edu.cn (H. Qiu).

1. Introduction

The mean curvature flow (MCF) in Euclidean space (pseudo-Euclidean space resp.) is a one-parameter family of immersions $X_t = X(\cdot, t) : M^m \rightarrow \mathbb{R}^{m+n}$ (\mathbb{R}_n^{m+n} resp.) with the corresponding image $M_t = X_t(M)$ such that

$$\begin{cases} \frac{d}{dt}X(x, t) = H(x, t) & x \in M, \\ X(x, 0) = X(x), \end{cases} \quad (1.1)$$

is satisfied, here $H(x, t)$ is the mean curvature vector of M_t at $X(x, t)$ in \mathbb{R}^{m+n} (\mathbb{R}_n^{m+n} resp.).

M^m is said to be a self-shrinker in \mathbb{R}^{m+n} (spacelike self-shrinker in \mathbb{R}_n^{m+n} resp.) if it satisfies a quasi-linear elliptic system

$$H = -X^N, \quad (1.2)$$

which is an important class of solutions to (1.1), where X^N is the normal part of X .

We call M^m a translating soliton in \mathbb{R}^{m+n} if it satisfies

$$H = -v^N, \quad (1.3)$$

where H is the mean curvature vector of M and v is a fixed vector in \mathbb{R}^{m+n} with unit length and v^N denotes the orthogonal projection of v onto the normal bundle of M .

Self-similar solutions to the MCF play an important role in understanding the behavior of the flow since they often occur as singularities. The subject of self-shrinkers in Euclidean spaces are also closely related with the theory of minimal submanifolds (see e.g. [4,16]). There is a plenty of works on the classification and uniqueness problem for self-shrinkers and translating solitons in Euclidean spaces (see e.g. [1,22,30,28,23,16,9,27,8,19,20,31,13]).

On the other hand, there are many works on the rigidity problem for complete spacelike submanifolds. Calabi [7] proposed the rigidity problem for complete spacelike extremal hypersurfaces in Minkowski space \mathbb{R}_1^{m+1} . He proved that such hypersurfaces have to be hyperplanes when $m \leq 4$. Cheng–Yau [15] solved the problem for all m , in sharp contrast to the situation of Euclidean space. Later, Jost–Xin [24] generalized the results to higher codimensions.

In view of the above two aspects, it is then natural to study the corresponding rigidity problems for spacelike self-shrinkers. Chau–Chen–Yuan [9] and Huang–Wang [21] proved that any spacelike entire graphic Lagrangian self-shrinkers must be flat under the condition that the Hessian of the potential function is bounded below quadratically by different methods. Ding–Wang in [18] derived rigidity results for spacelike entire graphs under subexponential decay condition. Ding–Xin [19] showed that such Lagrangian self-shrinkers are flat by removing the additional condition in [9] and [21]. Some rigidity and

Download English Version:

<https://daneshyari.com/en/article/4665176>

Download Persian Version:

<https://daneshyari.com/article/4665176>

[Daneshyari.com](https://daneshyari.com)