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Weak solutions to degenerate complex Monge–Ampère flows II ☆

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A B S T R A C T

Studying the (long-term) behavior of the Kähler–Ricci flow on mildly singular varieties, one is naturally led to study weak solutions of degenerate parabolic complex Monge–Ampère equations.

The purpose of this article, the second of a series on this subject, is to develop a viscosity theory for degenerate complex Monge–Ampère flows on compact Kähler manifolds. Our general theory allows in particular to define and study the (normalized) Kähler–Ricci flow on varieties with canonical singularities, generalizing results of Song and Tian.

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0. Introduction

The study of the (long-term) behavior of the Kähler–Ricci flow on mildly singular varieties in relation to the Minimal Model Program was undertaken by J. Song and G. Tian [29,30] and it requires a theory of weak solutions for certain degenerate parabolic complex Monge–Ampère equations modeled on:

$$\frac{\partial \phi}{\partial t} + \phi = \log \frac{(dd^c \phi)^n}{V} \tag{0.1}$$

where V is volume form and ϕ a t-dependent Kähler potential on a compact Kähler manifold. The approach in [30] is to regularize the equation and take limits of the solutions of the regularized equation with uniform higher order estimates. But as far as the existence and uniqueness statements in [30] are concerned, we believe that a zeroth order approach would be both simpler and more efficient.

There is a well established pluripotential theory of weak solutions to elliptic complex Monge–Ampère equations, following the pioneering work of Bedford and Taylor [2,3] in the local case (domains in $\mathbb{C}^n$). A complementary viscosity approach has been developed only recently in [23,17,34,18] both in the local and the global case (compact Kähler manifolds).

Surprisingly no similar theory has ever been developed on the parabolic side. The most significant reference for a parabolic flow of plurisubharmonic functions on pseudoconvex domains is [20] but the flow studied there takes the form

$$\frac{\partial \phi}{\partial t} = ((dd^c \phi)^n)^{1/n} \tag{0.2}$$

which does not make sense in the global case. The purpose of this article, the second of a series on this subject, is to develop a viscosity theory for degenerate complex Monge–Ampère flows of the form (0.3).

This article focuses on solving this problem on compact Kähler manifolds, while its companion [19] concerns the local case (domains in $\mathbb{C}^n$). More precisely we study here the complex degenerate parabolic complex Monge–Ampère flows

$$e^{\dot{\varphi}_t + F(t,x,\varphi)} \mu(t,x) - (\omega_t + dd^c \varphi_t)^n = 0, \tag{0.3}$$

where

- $T \in]0, +\infty]$;
- $\omega = \omega(t,x)$ is a continuous family of semi-positive $(1,1)$-forms on X,