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Trace class criteria for Toeplitz and composition operators on small Bergman spaces ☆

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ABSTRACT

We characterize the Schatten class Toeplitz operators induced by a positive Borel measure on the unit disc and the reproducing kernel of the Bergman space A^2_ω, where ω is a radial weight satisfying the doubling property $\int_r^1 \omega(s)\,ds \le C\int_{\frac{1+r}{2}}^1 \omega(s)\,ds$. By using this, we describe the Schatten class composition operators. We also discuss basic properties of composition operators acting from A^p_ω to A^q_v.

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1. Introduction and main results

Let $\mathcal{H}(\mathbb{D})$ denote the space of all analytic functions in the unit disc $\mathbb{D} = \{z : |z| < 1\}$. If $0 < p < \infty$ and ω is a weight, i.e. a nonnegative integrable function on $\mathbb{D}$, the weighted Bergman space A^p_ω consists of $f \in \mathcal{H}(\mathbb{D})$ such that

$$\|f\|^p_{A^p_\omega} = \int_{\mathbb{D}} |f(z)|^p \omega(z)\, dA(z) < \infty,$$

where $dA(z) = \frac{dx\, dy}{\pi}$ is the normalized Lebesgue area measure on $\mathbb{D}$. As usual, A^p_α denotes the weighted Bergman space induced by the standard radial weight $(1-|z|^2)^\alpha$.

In this study we consider Bergman spaces A^p_ω induced by weights in the class $\widehat{\mathcal{D}}$ of the radial ω for which $\widehat{\omega}(r) = \int_r^1 \omega(s)\, ds$ satisfies $\widehat{\omega}(r) \le C\widehat{\omega}(\frac{1+r}{2})$. The standard radial weights admit this doubling property, but $\widehat{\mathcal{D}}$ contains also weights such that $\widehat{\omega}(r)$ decreases to zero much slower than any positive power of $1-r$. It is known that many finer function theoretic properties of A^p_ω for such ω are very different from those of A^p_α and, in particular, its harmonic analysis is similar to that of Hardy spaces in many aspects [17–19].

For any $\omega \in \widehat{\mathcal{D}}$, the norm convergence in A^2_ω implies the uniform convergence on compact subsets, and hence the point evaluations L_z are bounded linear functionals on A^2_ω. Therefore there exist reproducing kernels $B^\omega_z \in A^2_\omega$, with $\|L_z\| = \|B^\omega_z\|_{A^2_\omega}$, such that

$$L_z(f) = f(z) = \langle f, B^\omega_z\rangle_{A^2_\omega} = \int_{\mathbb{D}} f(\zeta)\, \overline{B^\omega_z(\zeta)}\, \omega(\zeta)\, dA(\zeta), \quad f \in A^2_\omega.$$

These kernels give rise to the Toeplitz operator

$$\mathcal{T}_\mu(f)(z) = \int_{\mathbb{D}} f(\zeta)\, \overline{B^\omega_z(\zeta)}\, d\mu(\zeta),$$

where μ is a positive Borel measure on $\mathbb{D}$, which is the primary object in this study. The operator $\mathcal{T}_\mu$, associated with the kernel of a standard weighted Bergman space A^2_α and a measure $d\mu = \varphi dA$, has been extensively studied since the seventies [2,11,26]. Luecking [9] was probably one of the first authors to consider $\mathcal{T}_\mu$ with measures as symbols. He characterized those μ for which $\mathcal{T}_\mu$ belongs to the Schatten–Von Neumann ideal $\mathcal{S}_p(A^2_\alpha)$. His approach works also on the Hardy space H^2 and, for a certain range of p, on the classical weighted Dirichlet spaces [9,13]. Since Toeplitz operators go hand in hand with several other operators, Luecking's method has turned out to be useful in subsequent research on concrete operator theory [9,27].

Our main result describes the positive Borel measures μ such that $\mathcal{T}_\mu$ belongs to $\mathcal{S}_p(A^2_\omega)$ when $0 < p < \infty$ and $\omega \in \widehat{\mathcal{D}}$. Throughout the proof we will use the norm given by the Littlewood–Paley identity