

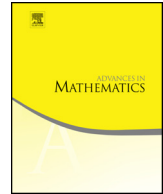


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Enriched categories as a free cocompletion[☆]Richard Garner^a, Michael Shulman^{b,1}^a Department of Computing, Macquarie University, North Ryde, NSW 2109, Australia^b Department of Mathematics, Institute for Advanced Study, Princeton, NJ 08540, USA

ARTICLE INFO

Article history:

Received 4 April 2013

Received in revised form 9 November 2015

Accepted 10 November 2015

Available online 28 November 2015

Communicated by Ross Street

Keywords:

Enriched bicategory theory

Enriched categories

Free cocompletions

Equipments

ABSTRACT

This paper has two objectives. The first is to develop the theory of bicategories enriched in a monoidal bicategory—categorifying the classical theory of categories enriched in a monoidal category—up to a description of the free cocompletion of an enriched bicategory under a class of weighted bicolimits. The second objective is to describe a universal property of the process assigning to a monoidal category \mathcal{V} the equipment of \mathcal{V} -enriched categories, functors, transformations, and modules; we do so by considering, more generally, the assignation sending an equipment \mathcal{C} to the equipment of \mathcal{C} -enriched categories, functors, transformations, and modules, and exhibiting this as the free cocompletion of a certain kind of enriched bicategory under a certain class of weighted bicolimits.

© 2015 Elsevier Inc. All rights reserved.

[☆] The first author acknowledges the support of Australian Research Council Discovery Project grants DP110102360 and DP130101969. The second author acknowledges the support of a United States National Science Foundation Postdoctoral Fellowship and a grant under agreement No. DMS-1128155. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

E-mail addresses: richard.garner@mq.edu.au (R. Garner), mshulman@ias.edu, shulman@sandiego.edu (M. Shulman).

¹ Current affiliation: Department of Mathematics and Computer Science, University of San Diego, San Diego, CA 92110, USA.

Contents

1.	Introduction	2
2.	Preliminaries	10
3.	\mathcal{V} -bicategories and their morphisms	13
4.	The tricategory of \mathcal{V} -bicategories	18
5.	Modules	21
6.	Tensor product of modules	29
7.	Internal hom of modules	34
8.	The Yoneda lemma	40
9.	\mathcal{V} -categories of right modules	43
10.	Colimits and left Kan extensions	47
11.	Colimits in categories of right modules	53
12.	Free cocompletions	57
13.	Change of base	59
14.	Constructions on monoidal bicategories	67
15.	Enriched categories and modules as a free cocompletion	70
16.	Enriched categories, functors, and modules as a free cocompletion	79
	References	93

1. Introduction

The classical theory of categories enriched in a monoidal category [18] has many applications throughout mathematics. The more general notion of a category enriched in a bicategory is less well-known, but it allows one to capture also *internal* categories and *indexed* categories through enrichment, and has been used in the study of sheaves and stacks [38,3,36]. More generally still, we can enrich categories in a double category or a proarrow equipment [39,26]; the advantage of this over bicategory-enrichment is a better notion of enriched functor (see [31,9] for some examples).

In this paper we do two things:

- (1) We categorify the theory of enriched categories to a theory of *bicategories* enriched in a monoidal bicategory, or more generally in a tricategory.
- (2) We show that the construction “ $\mathcal{C} \mapsto$ categories enriched in \mathcal{C} ”, for a bicategory or equipment \mathcal{C} , has a *universal property*.

While these objectives are perhaps seemingly unrelated, in fact the former is necessary for the latter: the universal property of enriched categories is expressed as a free cocompletion of a certain kind of enriched bicategory. This can be regarded as an instance of what Baez and Dolan [1] term the *microcosm principle*: the proper context in which to consider the theory of enriched categories is a categorified version of itself.

We now discuss (1) and (2) separately in somewhat more detail, beginning with (1).

Download English Version:

<https://daneshyari.com/en/article/4665215>

Download Persian Version:

<https://daneshyari.com/article/4665215>

[Daneshyari.com](https://daneshyari.com)