Advances in Mathematics 289 (2016) 480–566

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Adjunctions and defects in Landau–Ginzburg models

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ARTICLE INFO

Article history:
Received 27 May 2014
Received in revised form 6 March 2015
Accepted 23 March 2015
Available online 7 December 2015
Communicated by Henning Krause

To Ragnar-Olaf Buchweitz on the occasion of his sixtieth birthday

MSC:
18D05
57R56

Keywords:
Topological field theory
Bicategories
Matrix factorisations
Landau–Ginzburg models

ABSTRACT

We study the bicategory of Landau–Ginzburg models, which has polynomials as objects and matrix factorisations as 1-morphisms. Our main result is the existence of adjoints in this bicategory and formulas for the evaluation and coevaluation maps in terms of Atiyah classes and homological perturbation. The bicategorical perspective offers a unified approach to Landau–Ginzburg models: we show how to compute arbitrary correlators and recover the full structure of open/closed TFT, including the Kapustin–Li disc correlator and a simple proof of the Cardy condition, in terms of defect operators which in turn are directly computable from the adjunctions.

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http://dx.doi.org/10.1016/j.aim.2015.03.033
0001-8708/

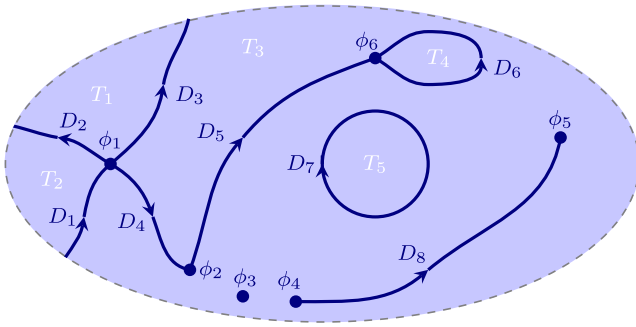

Fig. 1. Part of a worldsheet with defect lines and field insertions.

1. Introduction

Landau–Ginzburg models play an important role in many areas of mathematical physics and pure mathematics including singularity theory, representation theory, (homological) mirror symmetry, knot invariants, and conformal or topological field theory. The interplay between these areas is one of the aesthetic motivations for studying Landau–Ginzburg models. Another general motivation is their dual nature of affording insight into deep structure while being concrete enough to allow for hands-on computations.

In this paper we will show how this dichotomy manifests itself in the context of two-dimensional topological field theory (TFT) with defects. We explain how Landau–Ginzburg models give rise to a bicategory with adjoints (also called duals) and we describe the structure maps in this bicategory, which include the units and counits of adjunction (also called evaluation and coevaluation maps) in terms of basic invariants called Atiyah classes [2]. On the one hand this gives a satisfying explanation for duality in the setting of Landau–Ginzburg models in terms of commutation relations for Atiyah classes, and on the other hand it provides an effective way of evaluating arbitrary string diagrams in the bicategory. Since string diagrams in a bicategory can be identified with correlators in TFTs with defects, this opens the door for many applications.

In order to set the stage, and in particular explain how string diagrams are related to correlators, we recall a few aspects of TFTs with defects in an informal fashion; for more detailed accounts see [32] and [19, Section 2]. We imagine *bulk sector theories* T_I to "live" on a two-dimensional surface called the *worldsheet*. More precisely, the worldsheet may be partitioned into various domains to which the (not necessarily distinct) theories T_I are associated, and which are separated by one-dimensional oriented *defect lines* D_α. A sketch of a typical worldsheet is shown in Fig. 1.

In addition to the labels T_I for the two-dimensional domains and D_α for the one-dimensional defect lines, we also include labels ϕ_i for zero-dimensional points. These labels are interpreted as describing *fields* inserted at the points on the worldsheet. Note that the fields can also be placed at junctions of multiple defect lines.