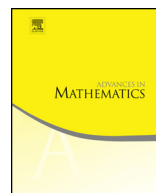




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Advances in Mathematics

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Green polynomials of Weyl groups, elliptic pairings, and the extended Dirac index [☆]

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ARTICLE INFO

Article history:

Received 25 February 2014

Received in revised form 10

December 2014

Accepted 6 July 2015

Available online 24 July 2015

Communicated by Roman

Bezrukavnikov

Keywords:

Weyl groups

Springer representations

Elliptic pairings

Dirac index

ABSTRACT

In this paper, we give a uniform construction of irreducible genuine characters of the Pin cover \widetilde{W} of a Weyl group W , and put them into the context of theory of Springer representations. In the process, we provide a direct connection between Springer theory, via Green polynomials, the irreducible representations of \widetilde{W} , and an extended Dirac operator for graded Hecke algebras. We also introduce a \mathbf{q} -elliptic pairing for W with respect to the reflection representation V . These constructions are of independent interest. The \mathbf{q} -elliptic pairing is a generalization of the elliptic pairing of W introduced by Reeder, and it is also related to S. Kato's

[☆] The authors thank George Lusztig for his suggestions regarding δ -quasidistinguished nilpotent elements and Zhiwei Yun for his suggestions regarding the $W_{\#}$ action on the cohomology of Springer fibers. The authors also thank Syu Kato for helpful discussions about Kostka systems, Roman Bezrukavnikov for pointing out the reference [7], and Eric Sommers for the reference [35]. The authors thank the referee for useful comments. This research was supported in part by the NSF grants DMS-0968065 and DMS-1302122, NSA-AMS 111016, and by the HKRGC grant 602011.

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notion of (graded) Kostka systems for the semidirect product
 $A_W = \mathbb{C}[W] \ltimes S(V)$.

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1. Introduction

1.1. Graded affine Hecke algebras were defined by Lusztig [22] in his study of representations of reductive p -adic groups and Iwahori–Hecke algebras. A Dirac operator \mathcal{D} for graded affine Hecke algebras was defined in [3], and, by analogy with the setting of Dirac theory for (\mathfrak{g}, K) -modules of real reductive groups, the notion of Dirac cohomology was introduced. The Dirac cohomology and the Dirac index in the Hecke algebra setting were further studied in [10,11]. The Dirac cohomology spaces are representations for a certain double cover (“pin cover”) \widetilde{W} of the Weyl group W . The irreducible representations of \widetilde{W} had been classified case by case in the work of Schur, Morris, Read and others, see for example [26,28,40]. Recently, it was noticed in [9] (again case by case) that there is a close relation between the representation theory of \widetilde{W} and the geometry of the nilpotent cone in semisimple Lie algebras \mathfrak{g} .

1.2. In this paper, we provide a direct link between:

- (a) an (extended) Springer W -action on cohomology groups;
- (b) the irreducible representations of \widetilde{W} , and
- (c) an (extended) Dirac index for tempered modules of graded Hecke algebras.

Here in (a) and (c), there is a canonical $\mathbb{Z}/2\mathbb{Z}$ -action, induced from the conjugation of the longest Weyl group element w_0 on W . The $\mathbb{Z}/2\mathbb{Z}$ -action looks irrelevant to the $\mathbb{Z}/2\mathbb{Z}$ -covering $\widetilde{W} \rightarrow W$. However, the remarkable thing is that one gets many more representations of \widetilde{W} with nontrivial extended Dirac index by using the twist given by such $\mathbb{Z}/2\mathbb{Z}$ -action. One of the main results (see Theorem 1.2 below) shows that all the irreducible genuine \widetilde{W} -representations (i.e., those that do not factor through W) appear in this way. This leads to a new, uniform construction of the irreducible genuine \widetilde{W} -characters.

1.3. To describe the role of this $\mathbb{Z}/2\mathbb{Z}$ -action, let us first make a short digression on elliptic representation theory.

Elliptic representation theory, introduced by Arthur [1], studies the Grothendieck group of certain representations of a Lie-theoretic group modulo those induced from proper parabolic subgroups. The elliptic theory of representations of semisimple p -adic groups and Iwahori–Hecke algebras was further studied intensively, e.g., Schneider–Stuhler [32], Bezrukavnikov [6], Reeder [29], Opdam–Solleveld [27]. In [29], Reeder also described the elliptic representations of W in terms of Springer correspondence. He proved an orthogonality result of Springer representations and proved that it is exactly

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