

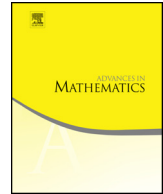


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



The extension algebra of some cohomological Mackey functors

Serge Bouc^a, Radu Stancu^{b,*}^a CNRS – LAMFA, CNRS UMR 7352, Université de Picardie, 33, rue St Leu, 80039, Amiens, France^b LAMFA, CNRS UMR 7352, Université de Picardie, 33, rue St Leu, 80039, Amiens, France

ARTICLE INFO

Article history:

Received 30 April 2014

Received in revised form 1 June 2015

Accepted 25 June 2015

Available online 24 July 2015

Communicated by Henning Krause

MSC:

18A25

18G10

18G15

20J05

Keywords:

Cohomological

Mackey functor

Extension

Simple

ABSTRACT

Let k be a field of characteristic p . We construct a new inflation functor for cohomological Mackey functors for finite groups over k . Using this inflation functor, we give an explicit presentation of the graded algebra of self-extensions of the simple functor S_1^G , when p is odd and G is an elementary abelian p -group.

© 2015 Elsevier Inc. All rights reserved.

1. Introduction

Let k be a field and G be a finite group. The theory of *Mackey functors* and *cohomological Mackey functors* for G over k originates in the work of Green [5] and Dress [4],

* Corresponding author.

E-mail addresses: serge.bouc@u-picardie.fr (S. Bouc), radu.stancu@u-picardie.fr (R. Stancu).

at the beginning of the 70s. It can be viewed as the theory of induction and restriction, when we forget the particular framework of linear representations of G . Many important developments have been achieved since, culminating in the comprehensive and seminal paper by Thévenaz and Webb [6] in 1995, where the authors introduced *the Mackey algebra* $\mu_k(G)$, and showed, among many other fundamental results, that the category of Mackey functors for G over k is equivalent to the category of $\mu_k(G)$ -modules. Similarly, they showed that the subcategory $\mathbf{M}_k^c(G)$ of cohomological Mackey functors for G over k is equivalent to the category of $\text{co}\mu_k(G)$ -modules, where $\text{co}\mu_k(G)$ is a specific quotient of $\mu_k(G)$, called *the cohomological Mackey algebra*.

The algebras $\mu_k(G)$ and $\text{co}\mu_k(G)$ share many similarities with the group algebra kG : e.g., they are finite dimensional k -vector spaces, of dimension independent of k , the Maschke theorem holds, there is a good theory of decomposition from characteristic 0 to characteristic p , etc. These resemblances raise a natural question, whether a given theorem on kG will admit an analogue for $\mu_k(G)$ or $\text{co}\mu_k(G)$.

This was the main motivation in [2], where the question of complexity of cohomological Mackey functors was solved (in the only non-trivial case where k is a field of positive characteristic p dividing the order of G). This question amounts to computing all extension groups between simple cohomological Mackey functors for G . It was shown in [2] that one can assume that G is a p -group, and in this case these extension groups can be determined from the knowledge of sufficient information on the algebra $\mathcal{E} = \text{Ext}^*(S_1^E, S_1^E)$ of self-extensions of a particular simple functor S_1^E for some subquotients E of G . Along the way, a presentation of this algebra when E is elementary abelian and $p = 2$ was given, together with a formula for the Poincaré series. In the case $p > 2$, no such presentation was given, and a conjecture was proposed for the Poincaré series of \mathcal{E} . In the same article, the conjecture was proved in the case $p = 3$.

This paper settles completely the case $p > 2$: a presentation of \mathcal{E} is given, and, as a corollary, the forementioned conjecture is proved. The main results are the following, where $S_{1,W}^H$ denotes the simple functor for the group H defined as in 2.15. To simplify notation, when $W = k$ is the trivial module, we drop this subscript. We start by the construction of a new inflation functor for cohomological Mackey functors:

1.1. Theorem. *Let k be a field, let G be a finite group, let $N \trianglelefteq G$ and let V be a simple $k(G/N)$ -module. Then there exists an exact functor $\sigma_{G/N}^G$ from $\mathbf{M}_k^c(G/N)$ to $\mathbf{M}_k^c(G)$ satisfying*

$$\sigma_{G/N}^G(S_{1,V}^{G/N}) = S_{1, \text{Inf}_{G/N}^G V}^G \quad .$$

Suppose, moreover, that $G = N \rtimes H$ is the semidirect product of N by a group H .

1. *If V is a kH -module, let \tilde{V} be the kG -module $\text{Inf}_{G/N}^G \text{Iso}_H^{G/N} V$. Then the restriction of \tilde{V} to H is isomorphic to V .*
2. *The composition $\text{Res}_H^G \sigma_{G/N}^G \text{Iso}_H^{G/N}$ is isomorphic to the identity functor of $\mathbf{M}_k^c(H)$.*

Download English Version:

<https://daneshyari.com/en/article/4665254>

Download Persian Version:

<https://daneshyari.com/article/4665254>

[Daneshyari.com](https://daneshyari.com)