

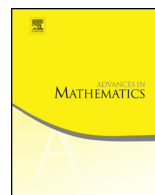


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On subordination of holomorphic semigroups <sup>☆</sup>Alexander Gomilko <sup>a,b</sup>, Yuri Tomilov <sup>c,\*</sup>

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## ABSTRACT

We prove that for any Bernstein function  $\psi$  the operator  $-\psi(A)$  generates a bounded holomorphic  $C_0$ -semigroup  $(e^{-t\psi(A)})_{t \geq 0}$  on a Banach space, whenever  $-A$  does. This answers a question posed by Kishimoto and Robinson. Moreover, giving a positive answer to a question by Berg, Boyadzhiev and de Laubenfels, we show that  $(e^{-t\psi(A)})_{t \geq 0}$  is holomorphic in the holomorphy sector of  $(e^{-tA})_{t \geq 0}$ , and if  $(e^{-tA})_{t \geq 0}$  is sectorially bounded in this sector then  $(e^{-t\psi(A)})_{t \geq 0}$  has the same property. We also obtain new sufficient conditions on  $\psi$  in order that, for every Banach space  $X$ , the semigroup  $(e^{-t\psi(A)})_{t \geq 0}$  on  $X$  is holomorphic whenever  $(e^{-tA})_{t \geq 0}$  is a bounded  $C_0$ -semigroup on  $X$ . These conditions improve and generalize well-known results by Carasso–Kato and Fujita.

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## 1. Introduction

The present paper concerns operator-theoretic and function-theoretic properties of Bernstein functions and solves several notable problems which have been left open for some time.

Bernstein functions play a prominent role in probability theory and operator theory. One of their characterizations, also important for our purposes, says that a function  $\psi : (0, \infty) \rightarrow [0, \infty)$  is Bernstein if and only if there exists a vaguely continuous semigroup of subprobability Borel measures  $(\mu_t)_{t \geq 0}$  on  $[0, \infty)$  such that

$$e^{-t\psi(\lambda)} = \int_0^\infty e^{-\lambda s} \mu_t(ds), \quad \lambda > 0, \quad (1.1)$$

for all  $t \geq 0$ .

Let now  $(e^{-tA})_{t \geq 0}$  be a bounded  $C_0$ -semigroup on a (complex) Banach space  $X$  with generator  $-A$ . The relation (1.1) suggests a way to define a new bounded  $C_0$ -semigroup  $(e^{-tB})_{t \geq 0}$  on  $X$  in terms of  $(e^{-tA})_{t \geq 0}$  and a Bernstein function  $\psi$  as

$$e^{-tB} = \int_0^\infty e^{-sA} \mu_t(ds), \quad (1.2)$$

where  $(\mu_t)_{t \geq 0}$  is a semigroup of measures given by (1.1). Following (1.1), it is natural to define  $\psi(A) := B$ . As it will be revealed in Subsection 3.3 below, such a definition of  $\psi(A)$  goes far beyond formal notation and it respects some rules for operator functions called functional calculus.

The semigroup  $(e^{-t\psi(A)})_{t \geq 0}$  is subordinated to the semigroup  $(e^{-tA})_{t \geq 0}$  via a subordinator  $(\mu_t)_{t \geq 0}$ . The basics of subordination theory was set up by Bochner [4] and Phillips [27]. This approach to constructing semigroups is motivated by probabilistic applications, e.g. by the study of Lévy processes, but it has also significant value for PDEs as well. As a textbook example one may mention a classical result of Yosida expressing  $(e^{-tA^\alpha})_{t \geq 0}$ ,  $\alpha \in (0, 1)$ , in terms of  $(e^{-tA})_{t \geq 0}$  as in (1.2), see e.g. [33]. The essential feature of this example is that  $C_0$ -semigroups  $(e^{-tA^\alpha})_{t \geq 0}$  turn out to be necessarily holomorphic. This fact stimulated further research on relations between functional calculi and Bernstein functions, see e.g. [31] and [32]. Some of them are described below.

An easy consequence of (1.2) is that for a fixed Bernstein function  $\psi$  the mapping

$$\mathcal{M} : -A \mapsto -\psi(A) \quad (1.3)$$

preserves the class of generators of bounded  $C_0$ -semigroups, and it is natural to ask whether there are any other important classes of semigroup generators stable under  $\mathcal{M}$ . In particular, whether  $\mathcal{M}$  preserves the class of holomorphic  $C_0$ -semigroups. The question

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