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Kolmogorov's problem on the class of multiply monotone functions

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ABSTRACT

In this paper we give necessary and sufficient conditions for the system of positive numbers $M_{k_1}, M_{k_2}, \dots, M_{k_d}$, $0 \le k_1 < \dots < k_d \le r$, to guarantee the existence of an r-monotone function defined on the negative half-line $\mathbb{R}_-$ and such that $\|x^{(k_i)}\|_\infty = M_{k_i}$, $i = 1, 2, \dots, d$. We also discuss some applications of the obtained results and connections with other problems.

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Hermite–Birkhoff interpolation
Extremal problems

1. Introduction

Many problems in Analysis are related to studying the necessary and sufficient conditions that guarantee existence of a function from a given class with a given set of characteristics, and to investigating the properties of the set of all functions with given characteristics. Interpolation of prescribed values at given points by functions from certain class (for example by polynomials, splines, perfect splines, etc.), Hermite and Birkhoff interpolation, the moment problem, and the Nevanlinna–Pick problem are all problems of such type. Note that there exist many versions of the moment problem (some will be mentioned later in this paper), that have been studied for over a hundred years and that, in turn, have connections and applications in many areas of Analysis, Probability, Statistics, and other areas of Mathematics.

Kolmogorov's problem about necessary and sufficient conditions to guarantee the existence of a function from a given class, which has prescribed values of norms of derivatives of given orders, can be considered as another problem of this type. It is likely that this question was motivated by Kolmogorov's work on the problem of dependence between norms of consecutive derivatives (problem on inequalities between norms of derivatives), that first appeared in the works of Hadamard, Hardy and Littlewood, and Landau. The latter question in turn has numerous applications in various branches of Analysis and has been an active area of research.

In spite of much effort no complete solution (for rather wide class of functions and arbitrary finite number of norms of derivatives) to Kolmogorov's problem has been found until now. In this paper we present a complete solution to Kolmogorov's problem for the class of multiply monotone functions and give its applications to some versions of the moment problem, the problem of the smoothest Hermite–Birkhoff interpolation, the Hermite–Birkhoff interpolation by perfect splines (with free knots and higher derivatives that take only two values, one of which is zero), and extremal properties of interpolating splines. We also consider some extremal problems on the sets of solutions of Kolmogorov's problem.

The paper is organized as follows. Section 2 presents the precise statement of Kolmogorov's problem, cases of the known solutions, and discusses what could naturally be considered as a solution to general Kolmogorov's problem. Section 3 contains necessary definitions and statements of the main results, which provide the answer to Kolmogorov's problem for arbitrary d positive numbers to be values of norms of prescribed derivatives of an r-monotone function from $L^r_{\infty,\infty}(\mathbb{R}_-)$. Section 4 is dedicated to the proofs of several supporting results which, when combined, constitute the proof of the main result of the paper. In Section 5 we discuss some connections between Kolmogorov's problem and some other problems of Analysis and Probability.