

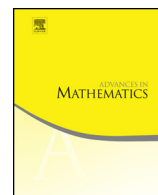


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Kernel estimates for nonautonomous Kolmogorov equations [☆]



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ABSTRACT

Using time dependent Lyapunov functions, we prove pointwise upper bounds for the heat kernels of some nonautonomous Kolmogorov operators with possibly unbounded drift and diffusion coefficients.

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1. Introduction

We establish kernel estimates for transition probabilities associated with nonautonomous evolution equations

$$\begin{cases} \partial_t u(t, x) = \mathcal{A}(t)u(t, x), & (t, x) \in (s, 1] \times \mathbb{R}^d, \\ u(s, x) = f(x), & x \in \mathbb{R}^d, \end{cases} \quad (1.1)$$

where the operators $\mathcal{A}(t)$ are defined on smooth functions φ by

$$(\mathcal{A}(t)\varphi)(x) = \sum_{ij=1}^d q_{ij}(t, x) D_{ij} \varphi(x) + \sum_{i=1}^d F_i(t, x) D_i \varphi(x),$$

and $s \in [0, 1)$. Parabolic equations like (1.1) appear naturally in connection with stochastic differential equations (cf. [26]), where it is natural to allow the coefficients to depend on time. Also in the study of Navier–Stokes flow past a rotating obstacle with Oseen condition, a suitable change of coordinates leads to a special class of nonautonomous Kolmogorov equations (cf. [12] and the references therein).

Throughout, we make the following assumptions on the coefficients.

Hypothesis 1.1. The coefficients q_{ij} , F_j ($i, j = 1, \dots, d$) are defined on $[0, 1] \times \mathbb{R}^d$ and

- (1) there exists an $\varsigma \in (0, 1)$ such that $q_{ij}, F_j \in C_{\text{loc}}^{\frac{\varsigma}{2}, \varsigma}([0, 1] \times \mathbb{R}^d)$ for all $i, j = 1, \dots, d$. Moreover, $q_{ij} \in C^{0,1}((0, 1) \times \mathbb{R}^d)$;
- (2) the matrix $Q = (q_{ij})$ is symmetric and uniformly elliptic in the sense that there exists a number $\eta > 0$ such that

$$\sum_{i,j=1}^d q_{ij}(t, x) \xi_i \xi_j \geq \eta |\xi|^2 \quad \text{for all } \xi \in \mathbb{R}^d, (t, x) \in [0, 1] \times \mathbb{R}^d;$$

- (3) there exist a nonnegative function $V \in C^2(\mathbb{R}^d)$ and a constant $M \geq 0$ such that $\lim_{|x| \rightarrow \infty} V(x) = \infty$ and we have $\mathcal{A}(t)V(x) \leq M$, as well as $\eta \Delta V(x) + F(t, x) \cdot \nabla V(x) \leq M$, for all $(t, x) \in [0, 1] \times \mathbb{R}^d$.

Note that neither q_{ij} nor F_j ($i, j = 1, \dots, d$) are assumed to be bounded in \mathbb{R}^d .

Under Hypothesis 1.1, it was proved in [15] that equation (1.1) is well posed in the sense that, for every $f \in C_b(\mathbb{R}^d)$, there exists a unique function $u \in C_b([s, 1] \times \mathbb{R}^d) \cap C^{1,2}((s, 1] \times \mathbb{R}^d)$ such that (1.1) is satisfied. Moreover, there exists an evolution family $(G(t, s))_{t, s \in D} \subset \mathcal{L}(C_b(\mathbb{R}^d))$, where $D := \{(t, s) \in [0, 1]^2 : t \geq s\}$, such that the unique solution u to (1.1) is given by $u = G(\cdot, s)f$. It turns out that each operator $G(t, s)$ is a contraction. We recall that an *evolution family* is a family $(G(t, s))_{(t, s) \in D}$ such that $G(t, t) = \text{id}_{C_b(\mathbb{R}^d)}$ and, for $r, s, t \in [0, 1]$ with $r \leq s \leq t$, the *evolution law* $G(t, s)G(s, r) =$

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