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Association schemes on general measure spaces and zero-dimensional Abelian groups

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ABSTRACT

Association schemes form one of the main objects of algebraic combinatorics, classically defined on finite sets. At the same time, direct extensions of this concept to infinite sets encounter some problems even in the case of countable sets, for instance, countable discrete Abelian groups. In an attempt to resolve these difficulties, we define association schemes on arbitrary, possibly uncountable sets with a measure. We study operator realizations of the adjacency algebras of schemes and derive simple properties of these algebras. However, constructing a complete theory in the general case faces a set of obstacles related to the properties of the adjacency algebras and associated projection operators. To develop a theory of association schemes, we focus on schemes on topological Abelian groups where we can employ duality theory and the machinery of harmonic analysis. Using the language of spectrally dual partitions, we prove that such groups support the construction of general Abelian (translation) schemes and establish properties of their spectral parameters (eigenvalues). Addressing the existence question of spectrally dual partitions, we show that they arise naturally on topological zero-

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dimensional Abelian groups, for instance, Cantor-type groups or the groups of p-adic numbers. This enables us to construct large classes of examples of dual pairs of association schemes on zero-dimensional groups with respect to their Haar measure, and to compute their eigenvalues and intersection numbers (structural constants). We also derive properties of infinite metric schemes, connecting them with the properties of the non-Archimedean metric on the group.

Next we focus on the connection between schemes on zero-dimensional groups and harmonic analysis. We show that the eigenvalues have a natural interpretation in terms of Littlewood–Paley wavelet bases, and in the (equivalent) language of martingale theory. For a class of nonmetric schemes constructed in the paper, the eigenvalues coincide with values of orthogonal function systems on zero-dimensional groups. We observe that these functions, which we call Haar-like bases, have the properties of wavelet bases on the group, including in some special cases the self-similarity property. This establishes a seemingly new link between algebraic combinatorics and (non-Archimedean) harmonic analysis.

We conclude the paper by studying some analogs of problems of classical coding theory related to the theory of association schemes.

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