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Compact Kähler manifolds admitting large solvable groups of automorphisms

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ABSTRACT

Let G be a group of automorphisms of a compact Kähler manifold X of dimension n and $N(G)$ the subset of null-entropy elements. Suppose G admits no non-abelian free subgroup. Improving the known Tits alternative, we obtain that, up to replace G by a finite-index subgroup, either $G/N(G)$ is a free abelian group of rank $\leq n-2$, or $G/N(G)$ is a free abelian group of rank $n-1$ and X is a complex torus, or G is a free abelian group of rank $n-1$. If the last case occurs, X is G-equivariant birational to the quotient of an abelian variety provided that X is a projective manifold of dimension $n \geq 3$ and is not rationally connected. We also prove and use a generalization of a theorem by Fujiki and Lieberman on the structure of $\mathrm{Aut}(X)$.

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1. Introduction

We work over the field $\mathbb{C}$ of complex numbers. Let X be a compact Kähler manifold of dimension n and $\mathrm{Aut}(X)$ the group of all (holomorphic) automorphisms of X. Fujiki and Lieberman proved that $\mathrm{Aut}(X)$ is a complex Lie group of finite dimension but it may have an infinite number of connected components. The identity connected component $\mathrm{Aut}_0(X)$ of $\mathrm{Aut}(X)$ is the set of automorphisms obtained by integrating holomorphic vector fields on X. It is a normal subgroup of $\mathrm{Aut}(X)$ and its action on the Hodge cohomology groups $H^{p,q}(X, \mathbb{C})$ is trivial. See [10,14] for details.

Elements in $\mathrm{Aut}_0(X)$ have zero (topological) entropy. Indeed, for any automorphism $g \in \mathrm{Aut}(X)$, the *entropy* of g, defined in the theory of dynamical systems, turns out to be equal to the logarithm of the spectral radius of the pull-back operator g^* acting on $\oplus_{0\leq p\leq n} H^{p,p}(X, \mathbb{R})$. This is a consequence of theorems due to Gromov and Yomdin. Topological entropy is always a non-negative number and also equals the logarithm of the spectral radius of g^* acting on the whole cohomology group $\oplus_{0\leq p,q\leq n} H^{p,q}(X, \mathbb{C})$. This number is strictly positive if and only if the spectral radius of g^* acting on $H^{p,p}(X, \mathbb{R})$ is strictly larger than 1 for all or for some p with $1 \leq p \leq n - 1$. See [5,12,17] for details.

The group $\mathrm{Aut}(X)$ satisfies the following Tits alternative type result which was proved in [3, Theorem 1.5] (generalizing [18, Theorem 1.1]). See [5] for a survey. Recall that a group H is *virtually solvable* (resp. *free abelian*, *unipotent*, ...), if a finite-index subgroup of H is solvable (resp. free abelian, unipotent, ...).

Theorem 1.1. *Let X be a compact Kähler manifold of dimension $n \geq 2$ and $G \leq \mathrm{Aut}(X)$ a group of automorphisms. Then one of the following two alternative assertions holds:*

(1) *G contains a subgroup isomorphic to the non-abelian free group $\mathbb{Z} * \mathbb{Z}$, and hence G contains subgroups isomorphic to non-abelian free groups of all countable ranks.*
(2) *G is virtually solvable.*

In the second case, or more generally, when the representation $G|_{H^2(X,\mathbb{C})}$ is virtually solvable, G contains a finite-index subgroup G_1 such that the **null-entropy subset**

$$N(G_1) := \{g \in G_1 \mid g \textit{ is of null entropy}\}$$

is a normal subgroup of G_1 and the quotient $G_1/N(G_1)$ is a free abelian group of rank $r \leq n - 1$.

When the action of G on $H^2(X, \mathbb{C})$ is virtually solvable, the integer r in the theorem is called *the dynamical rank* of G and we denote it as $r = r(G)$. It does not depend on the choice of G_1. When $G|_{H^2(X,\mathbb{C})}$ is abelian, Theorem 1.1 can be deduced from [6, Theorem 1] and the classical Tits alternative for linear algebraic groups. In [6, Remarque 4.9], the authors mentioned the interest of studying these X admitting a commutative G of positive entropy and maximal dynamical rank $n - 1$.