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The stabilized set of p's in Krivine's theorem can be disconnected [☆]

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ABSTRACT

For any closed subset F of $[1, \infty]$ which is either finite or consists of the elements of an increasing sequence and its limit, a reflexive Banach space X with a 1-unconditional basis is constructed so that in each block subspace Y of X, ℓ_p is finitely block represented in Y if and only if $p \in F$. In particular, this solves the question as to whether the stabilized Krivine set for a Banach space had to be connected. We also prove that for every infinite dimensional subspace Y of X there is a dense subset G of F such that the spreading models admitted by Y are exactly the ℓ_p for $p \in G$.

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Finite block representability
Krivine's theorem

1. Introduction

In the past, many of the driving questions in the study of Banach spaces concerned the existence of "nice" subspaces of general infinite dimensional Banach spaces. Finding counterexamples to these questions involved developing new ideas for constructing Banach spaces. B. Tsirelson's construction of a reflexive infinite dimensional Banach space which does not contain ℓ_p for any $1 < p < \infty$ [16] and W.T. Gowers and B. Maurey's construction of an infinite dimensional Banach space which does not contain an unconditional basic sequence [7] are two important examples. On the other hand, after Tsirelson's construction, J.-L. Krivine proved that every basic sequence contains ℓ_p for some $1 \leqslant p \leqslant \infty$ finitely block represented [9] (where the case $p = \infty$ refers to c_0), and it is not difficult to show that every normalized weakly null sequence in a Banach space has a subsequence with a 1-suppression unconditional spreading model. Thus, though we cannot always find these properties in infinite dimensional subspaces, they are still always present in certain finite block or asymptotic structure.

In his paper on Krivine's Theorem, Rosenthal proved that given any Banach space, the set of p's such that ℓ_p is finitely block represented in the Banach space can be stabilized on a subspace [14] (for a simplified proof of the stability result see also [10, page 133]). That is, given any infinite dimensional Banach space X, there exists an infinite dimensional subspace $Y \subseteq X$ with a basis and a nonempty closed subset $I \subseteq [1, \infty]$ such that for every block subspace Z of Y, ℓ_p is finitely block represented in Z if and only if $p \in I$. Rosenthal concluded his paper by asking if this stabilized Krivine set I had to be a singleton. E. Odell and Th. Schlumprecht answered this question by constructing a Banach space X with an unconditional basis which had the property that every unconditional basic sequence is finitely block represented in every block sequence in X [12]. Thus, the stabilized Krivine set for this space is the interval $[1, \infty]$. Later, Odell and Schlumprecht constructed a Banach space with a conditional basis which had the property that every monotone basic sequence is finitely block represented in every block sequence in X [13]. At this point, the known possible stabilized Krivine sets for a Banach space are singletons and the entire interval $[1, \infty]$. P. Habala and N. Tomczak-Jaegermann proved that if $1 \leqslant p < q \leqslant \infty$ and X is an infinite dimensional Banach space such that ℓ_p and ℓ_q are finitely block represented in every block subspace of X then X has a quotient Z so that every $r \in [p, q]$ is finitely block represented in Z [8]. They then asked if the stabilized Krivine set for a Banach space is always connected [8], which was later included as problem 12 in Odell's presentation of 15 open problems in Banach spaces at the Fields institute in 2002 [11]. We solve the stabilized Krivine set problem with the following theorem.