Advances in Mathematics 282 (2015) 443–515

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

On the instability problem of a 3-D transonic oblique shock wave ☆

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ARTICLE INFO

Article history:
Received 19 July 2014
Received in revised form 26 April 2015
Accepted 22 June 2015
Available online 24 July 2015
Communicated by Camillo De Lellis

MSC:
35L70
35L65
35L67
76N15

Keywords:
Supersonic flow
Potential equation
Transonic oblique shock
Modified Bessel function
Overdetermined
Unstable

ABSTRACT

In this paper, we are concerned with the instability problem of a 3-D transonic oblique shock wave for the steady supersonic flow past an infinitely long sharp wedge. The flow is assumed to be isentropic and irrotational. It was indicated on p. 317 of [7] that if a steady supersonic flow comes from minus infinity and hits a sharp symmetric wedge, then it follows from the Rankine–Hugoniot conditions and the physical entropy condition that there possibly appears a weak shock or a strong shock attached at the edge of the sharp wedge, which corresponds to a supersonic shock or a transonic shock, respectively. The question arises which of the two actually occurs. It has frequently been stated that the strong one is unstable and that, therefore, only the weak one could occur. However, a convincing proof of this instability has apparently never been given. The aim of this paper is to understand such a longstanding open question. We will show that the attached 3-D transonic oblique shock problem is overdetermined with

☆ Li Liang and Yin Huicheng are supported by the NSFC (No. 11025105) and a project funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions. Xu Gang is supported by the NSFC (Nos. 11101190, 11371189, 11271164).

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http://dx.doi.org/10.1016/j.aim.2015.06.018

respect to the periodic perturbation, which implies that the 3-D transonic shock is unstable in general.

1. Introduction

In this paper, we are concerned with the instability problem of a 3-D transonic oblique shock for the steady supersonic flow past an infinitely long sharp wedge (see Fig. 1). As indicated on p. 317 of [7]: if a supersonic steady flow comes from minus infinity and hits a sharp symmetric wedge, then it follows from the Rankine–Hugoniot conditions and the physical entropy condition that there will appear a weak shock or a strong shock attached at the edge of the sharp wedge, which corresponds to a supersonic shock or a transonic shock, respectively. The question arises which of the two shocks actually occurs. It has frequently been stated that the strong one is unstable and that, therefore, only the weak one could occur. However, a convincing proof of this instability has apparently never been given. The aim of this paper is to understand such a longstanding open question. With respect to the 2-D weak oblique shock, under some different assumptions on the 2-D sharp wedge, the authors in [17,21,30] have respectively established the local/global existence and stability of a supersonic shock solution for the perturbed supersonic incoming flow past a 2-D sharp curved wedge. For the 3-D weak oblique shock, Chen S.X. in [3] has shown its local stability. With respect to the 2-D strong oblique shock, under certain pressure condition at infinity in the downstream subsonic region, the authors in [4] and [31] have proved the global existence and stability of a transonic shock for the 2-D potential equation and the 2-D full Euler system respectively, which are contrary to the conjecture on the instability of the transonic oblique shock (this instability conjecture has been mentioned in the above). In addition, for the 2-D unsteady potential equation, the authors in [10] constructed a self-similar analytic solution which connects an attached 2-D strong shock with an attached 2-D weak shock when a supersonic flow hits a 2-D sharp wedge. Note that the realistic world is three-dimensional. The purpose of this paper is to show that the attached 3-D transonic shock problem is overdetermined for the periodic perturbation, which means that the 3-D transonic shock is unstable in general and therefore gives a rather positive illustration of the instability of a 3-D transonic oblique shock. This also indicates that the space dimensions are essential for answering the stability or instability of the transonic oblique shocks.

We will assume that the supersonic incoming flow is of a small perturbation with respect to the constant supersonic state $(\rho_0, q_0, 0, 0)$ and such a flow hits the sharp 3-D wedge $\{x : x_1 \geq 0, x_2 \in \mathbb{R}, -b_0x_1 \leq x_3 \leq b_0x_1\}$ along the x_1-direction (see Fig. 2). Due to the non-interaction property of the transonic oblique shocks on two sides of the wedge, then it suffices to consider our transonic shock problem only in the upper