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Strichartz estimates and nonlinear wave equation on nontrapping asymptotically conic manifolds

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A B S T R A C T

We prove the global-in-time Strichartz estimates for wave equations on the nontrapping asymptotically conic manifolds. We obtain estimates for the full set of wave admissible indices, including the endpoint. The key points are the properties of the microlocalized spectral measure of Laplacian on this setting showed in [18] and a Littlewood–Paley squarefunction estimate. As applications, we prove the global existence and scattering for a family of nonlinear wave equations on this setting.

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1. Introduction and statement of main results

Let (M°, g) be a Riemannian manifold of dimension $n \geq 2$, and let $I \subset \mathbb{R}$ be a time interval. Suppose $u(t,z)$: $I \times M^\circ \to \mathbb{C}$ to be the solutions of the wave equation

$$\partial_t^2 u + \mathrm{H}u = 0, \qquad u(0) = u_0(z), \qquad \partial_t u(0) = u_1(z)$$

where $\mathrm{H} = -\Delta_g$ denotes the minus Laplace–Beltrami operator on (M°, g). The general homogeneous Strichartz estimates read

$$\big\|u(t,z)\big\|_{L^q_t L^r_z(I\times M^\circ)} \leq C\big(\|u_0\|_{H^s(M^\circ)} + \|u_1\|_{H^{s-1}(M^\circ)}\big),$$

where H^s denotes the L^2-Sobolev space over M°, and $2 \leq q, r \leq \infty$ satisfy

$$s = n\left(\frac{1}{2} - \frac{1}{r}\right) - \frac{1}{q}, \qquad \frac{2}{q} + \frac{n-1}{r} \leq \frac{n-1}{2}, \qquad (q,r,n) \neq (2,\infty,3).$$

In the flat Euclidean space, where $M^\circ = \mathbb{R}^n$ and $g_{jk} = \delta_{jk}$, one can take $I = \mathbb{R}$; see Strichartz [28], Ginibre and Velo [10], Keel and Tao [20], and references therein. In general manifolds, for instance the compact manifold with or without boundary, most of the Strichartz estimates are local in time. If M° is a compact manifold without boundary, due to finite speed of propagation one usually works in coordinate charts and establishes local Strichartz estimates for variable coefficient wave operators on $\mathbb{R}^n$. See for examples [19,24,29]. Strichartz estimates also are considered on compact manifold with boundary, see [6,3] and references therein. When we consider the noncompact manifold with nontrapping condition, one can obtain global-in-time Strichartz estimates. For instance, when M° is an exterior manifold in $\mathbb{R}^n$ to a convex obstacle, for metrics g which agree with the Euclidean metric outside a compact set with nontrapping assumption, the global Strichartz estimates are obtained by Smith and Sogge [25] for odd dimension, and Burq [5] and Metcalfe [23] for even dimension. Blair, Ford and Marzuola [2] established global Strichartz estimates for the wave equation on flat cones $C(\mathbb{S}^1_\rho)$ by using the explicit representation of the fundamental solution.

In this paper, we consider the establishment of global-in-time Strichartz estimates on asymptotically conic manifolds satisfying a nontrapping condition. Here, 'asymptotically conic' is meant in the sense that M° can be compactified to a manifold with boundary M such that g becomes a scattering metric on M. On the nontrapping asymptotically conic manifolds, Hassell, Tao, and Wunsch first established an $L^4_{t,z}$-Strichartz estimate for Schrödinger equation in [14] and then they [15] extended the estimate to full admissible Strichartz exponents except endpoint $q = 2$. More precisely, they obtained the local-in-time Strichartz inequalities for non-endpoint Schrödinger admissible pairs (q, r)

$$\big\|e^{it\Delta_g}u_0\big\|_{L^q_t L^r_z([0,1]\times M^\circ)} \leq C\|u_0\|_{L^2(M^\circ)}.$$