

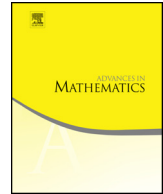


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Deformation quantization of Leibniz algebras

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ABSTRACT

In this paper, we use the local integration of a Leibniz algebra \mathfrak{h} using a Baker–Campbell–Hausdorff type formula in order to deformation quantize its linear dual \mathfrak{h}^* . More precisely, we define a natural rack product on the set of exponential functions on \mathfrak{h}^* which extends to a rack action on $C^\infty(\mathfrak{h}^*)$.

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0. Introduction

In this paper, we solve an old problem in symplectic geometry, namely we propose a way how to quantize the dual space of a Leibniz algebra \mathfrak{h} . This dual space \mathfrak{h}^* is some kind of generalized Poisson manifold, as the bracket of \mathfrak{h} is not necessarily skew-symmetric. Intimately linked to this question is the integration of Leibniz algebras.

In the search of understanding the periodicity in K-theory, J.-L. Loday introduced Leibniz algebras as non-commutative analogues of Lie algebras. More precisely, a real

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Leibniz algebra is a real vector space with a bracket which satisfies the (left) Leibniz identity

$$[X, [Y, Z]] = [[X, Y], Z] + [Y, [X, Z]],$$

but is not necessarily skew-symmetric. Leibniz algebras are a well-established algebraic structure generalizing Lie algebras (those Leibniz algebras where the bracket is skew-symmetric) with their own structure, deformation and homology theory. In the same way the Lie algebra homology of matrices (over a commutative ring containing the rational numbers) defines additive K-theory (i.e. cyclic homology), the Leibniz homology of matrices defines some non-commutative additive K-theory (in fact, Hochschild homology). Loday was mainly interested in the properties of the corresponding homology theory on “group level” (“Leibniz K-Theory”), and therefore asked the question: which (generalization of the structure of Lie groups) is the correct structure to integrate Leibniz algebras?

Kinyon [15] explored Lie racks as a structure integrating Leibniz algebras. Racks are roughly speaking an axiomatization of the structure of the conjugation in a group. The rack product on a group is simply given by

$$g \triangleright h := ghg^{-1},$$

and a general rack product on a set X is a binary operation satisfying for all $x, y, z \in X$ that $x \triangleright - : X \rightarrow X$ is bijective and the autodistributivity relation

$$x \triangleright (y \triangleright z) = (x \triangleright y) \triangleright (x \triangleright z).$$

Lie racks are the smooth analogues of racks. Kinyon showed (see [Theorem 1.25](#)) that the tangent space at the distinguished element 1 of a Lie rack carries in a natural way a Leibniz bracket. The idea is to differentiate twice the rack structure, mimicking exactly how the conjugation in a Lie group is differentiated to give first the map Ad , the adjoint action of the group on the Lie algebra, and then the Lie bracket in terms of ad , the adjoint action of the Lie algebra on itself. He did not judge racks to be the correct objects integrating Leibniz algebras. As a reason for this, he showed that all Leibniz algebras integrate into Lie racks, but in a kind of arbitrary way, as this integration does not appear to give Lie groups in case one started with a Lie algebra. It is clear (and useful as a guiding principle) that from this point of view, integrating Leibniz algebras means just an *integration of the adjoint action* of a Leibniz algebra on itself. From here stems the most important example for us of a rack product, namely

$$X \triangleright Y := e^{\text{ad}_X}(Y),$$

for all $X, Y \in \mathfrak{h}$ for a Leibniz algebra \mathfrak{h} .

On the other hand, Covez [10] showed in his 2010 doctoral thesis how to adapt the homological proof of Lie’s Third Theorem to Leibniz algebras. Regarding a given real

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