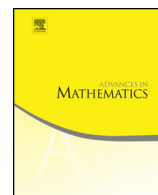




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## Decay of eigenfunctions of elliptic PDE's, I

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## ABSTRACT

We study exponential decay of eigenfunctions of self-adjoint higher order elliptic operators on  $\mathbb{R}^d$ . We show that the possible (global) critical decay rates are determined algebraically. In addition we show absence of super-exponentially decaying eigenfunctions and a refined exponential upper bound.

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## 1. Introduction and results

Consider a real elliptic polynomial  $Q$  of degree  $q$  on  $\mathbb{R}^d$ . We consider the operator  $H = Q(p) + V(x)$ ,  $p = -i\nabla$ , on  $L^2 = L^2(\mathbb{R}^d)$  with  $V$  real-valued, bounded and measurable and with  $\lim_{|x| \rightarrow \infty} V(x) = 0$ . By the assumptions on  $Q$  the operator  $Q(p)$  is self-adjoint on the standard Sobolev space of order  $q$  which consequently is the domain of  $H$  too. The goal of the paper is to study exponential decay of  $L^2$ -eigenfunctions of  $H$ . It is the first in a series of two papers on the subject, the second one is [\[10\]](#).

We will mostly assume there is a splitting of  $V$ ,  $V = V_1 + V_2$ , into real-valued bounded functions,  $V_1$  smooth and  $V_2$  measurable, with additional assumptions depending on the result. With virtually no complication of proofs  $V_2$  could be taken complex-valued.

For a given  $\lambda \in \mathbb{R}$  the energy surface

$$S_\lambda = \{(x, \xi) \in \mathbb{R}^d \times \mathbb{R}^d \mid Q(\xi) = \lambda\}$$

is by definition regular if  $\lambda$  is not a critical value of  $Q$ , that is if

$$\nabla Q(\xi) \neq 0 \quad \text{on } S_\lambda. \tag{1.1}$$

We will need this condition in one of our results.

Suppose  $(H - \lambda)\phi = 0$ ,  $\phi \in L^2$ . The *critical decay rate* is defined as

$$\sigma_c = \sup\{\sigma \geq 0 \mid e^{\sigma|x|}\phi \in L^2\}.$$

In this paper we shall study this notion of global decay rate for eigenfunctions, cf. previous works for the Laplacian [\[3,6–8\]](#) corresponding to the case  $Q(\xi) = \xi^2$ . We devote [\[10\]](#) to the study of the so-called local critical decay rate, see Subsection [1.3](#) for a definition and an announcement of some results of the second paper of this series. In particular we give in the present paper necessary (phase-space) conditions for a positive number  $\sigma$  to

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