

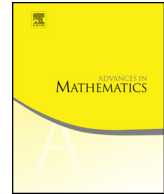


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# Existence of pulses in excitable media with nonlocal coupling <sup>☆</sup>



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## ABSTRACT

We prove the existence of fast traveling pulse solutions in excitable media with non-local coupling. Existence results had been known, until now, in the case of local, diffusive coupling and in the case of a discrete medium, with finite-range, non-local coupling. Our approach replaces methods from geometric singular perturbation theory, that had been crucial in previous existence proofs, by a PDE oriented approach, relying on exponential weights, Fredholm theory, and commutator estimates.

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## 1. Introduction

Excitable media play a central role in our understanding of complex systems. Chemical reactions [2,19], calcium waves [34], and neural field models [6,7] are among the examples that motivate our present study. A prototypical model of excitable kinetics are

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the FitzHugh–Nagumo kinetics, derived first as a simplification of the Hodgkin–Huxley model for the propagation of electric signals through nerve fibers [20],

$$\frac{du}{dt} = f(u) - v, \quad (1.1a)$$

$$\frac{dv}{dt} = \epsilon(u - \gamma v), \quad (1.1b)$$

where, for instance,  $f(u) = u(1-u)(u-a)$ . For  $0 < a < 1/2$  and  $\gamma > 0$ , not too large, all trajectories in this system converge to the trivial equilibrium  $u = v = 0$ . The system is however excitable in the sense that finite-size perturbations of  $u$ , past the excitability threshold  $a$ , away from the stable equilibrium  $u = v = 0$ , can induce a long transient, where  $f(u) \sim v$ ,  $u > 1/2$ . During these transients, which last for times  $\mathcal{O}(1/\epsilon)$ ,  $u$  is said to be in the excited state; eventually,  $u$  returns to values  $f(u) \sim v$ ,  $u < 1/2$ , the quiescent state.

Interest in these systems stems from the fact that, although kinetics are very simple and ubiquitous in nature, with convergence of all trajectories to a simple stable equilibrium, spatial coupling can induce quite complex dynamics. The simplest example is the propagation of a stable excitation pulse, more complicated examples include two-dimensional spiral waves and spatio-temporal chaos. Intuitively, a local excitation can trigger excitations of neighbors before decaying back to the quiescent state in a spatially coupled system. After initial transients, one then observes a spatially propagating region where  $u$  belongs to the excited state.

Rigorous approaches to the existence of such excitation pulses have been based on singular perturbation methods. Consider, for example,

$$\partial_t \mathbf{u}(x, t) = \partial_{xx} \mathbf{u}(x, t) + f(\mathbf{u}(x, t)) - \mathbf{v}(x, t), \quad (1.2a)$$

$$\partial_t \mathbf{v}(x, t) = \epsilon(\mathbf{u}(x, t) - \gamma \mathbf{v}(x, t)) \quad (1.2b)$$

with  $x \in \mathbb{R}$ . One looks for solutions of the form

$$(\mathbf{u}, \mathbf{v})(x, t) = (\mathbf{u}, \mathbf{v})(x - ct), \quad (1.3)$$

and finds first-order ordinary differential equation for  $\mathbf{u}, \mathbf{u}_x, \mathbf{v}$ , in which one looks for a homoclinic solution to the origin. The small parameter  $\epsilon$  introduces a singularly perturbed structure into the problem which allows one to find such a homoclinic orbit by tracking stable and unstable manifolds along fast intersections and slow, normally hyperbolic manifolds [9,18,24]. This approach has been successfully applied in many other contexts with slow-fast like structures, with higher- or even infinite-dimensional slow-fast ODEs; see for instance [22,23,36].

Our interest is in media with *infinite-range* coupling. We will focus on *linear* coupling through convolutions, although we believe that the existence result extends to a variety of other problems. To fix ideas, we consider

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