

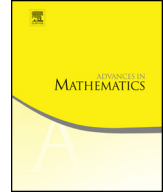


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Advances in Mathematics

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Global existence for some transport equations with nonlocal velocity



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ARTICLE INFO

Article history:

Received 21 May 2014

Accepted 22 October 2014

Available online 31 October 2014

Communicated by Charles Fefferman

This paper is dedicated to Professor Eitan Tadmor on the occasion of his 60th birthday

Keywords:

Transport equation

Nonlocal velocity field

Weak solution

Entropy

ABSTRACT

In this paper, we study transport equations with nonlocal velocity fields with rough initial data. We address the global existence of weak solutions of a one dimensional model of the surface quasi-geostrophic equation and the incompressible porous media equation, and one dimensional and n dimensional models of the dissipative quasi-geostrophic equations and the dissipative incompressible porous media equation.

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1. Introduction

In this paper, we study several active scalar equations with nonlocal velocity fields. Here, the non-locality means that the velocity field is defined through a singular integral operator that is represented in terms of a Fourier multiplier. For example, in the

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two dimensional Euler equation in vorticity form [6], the velocity is recovered from the vorticity ω through

$$u = \nabla^\perp(-\Delta)^{-1}\omega \quad \text{or equivalently} \quad \widehat{u}(\xi) = \frac{i\xi^\perp}{|\xi|^2}\widehat{\omega}(\xi). \tag{1.1}$$

Other nonlocal and quadratically nonlinear equations appear in many applications. Prototypical examples are the surface quasi-geostrophic equation, the incompressible porous medium equation, Stokes equations, magnetogeostrophic equation and their variants. We briefly introduce the equations below.

The surface quasi-geostrophic equations. The surface quasi-geostrophic equation describes the dynamics of the mixture of cold and hot air and the fronts between them in 2 dimensions [23,51]. The equation is of the form

$$\theta_t + u \cdot \nabla\theta = 0, \quad u = (-\mathcal{R}_2\theta, \mathcal{R}_1\theta), \tag{1.2}$$

where the scalar function θ is the potential temperature and \mathcal{R}_j is the Riesz transform

$$\mathcal{R}_j f(x) = \frac{1}{2\pi} \text{p.v.} \int_{\mathbb{R}^2} \frac{(x_j - y_j)f(y)}{|x - y|^3} dy, \quad j = 1, 2.$$

Similar model equations of (1.2) with different types of nonlocal velocities are proposed and analyzed in [2,9,19], respectively (see also [4,10,18,40,44,42,43]):

$$\begin{aligned} \theta_t + u \cdot \nabla\theta &= 0, \quad u = \mathcal{R}\theta, \\ \theta_t + \nabla \cdot (\theta\mathcal{R}\theta) &= 0, \\ \theta_t + u \cdot \nabla\theta &= 0, \quad u = \nabla^\perp A^{\beta-2}\theta, \quad 1 < \beta \leq 2. \end{aligned}$$

We finally introduce the two dimensional Euler- α model in vorticity form:

$$\theta_t + u \cdot \nabla\theta = 0, \quad u = \nabla^\perp A^{-2+\alpha}\theta, \quad \alpha \in [0, 1] \tag{1.3}$$

which interpolates between (1.1) ($\alpha = 0$) and (1.2) ($\alpha = 1$).

The incompressible porous medium equation. This equation takes the form [17]

$$\theta_t + u \cdot \nabla\theta = 0, \quad u = \mathcal{R}^\perp \mathcal{R}_1\theta, \tag{1.4}$$

where θ is now the density of the incompressible fluid moving through a homogeneous porous medium. The following version also has been studied with $\beta > 0$ [31]:

$$\theta_t + u \cdot \nabla\theta = 0, \quad u = A^\beta \mathcal{R}^\perp \mathcal{R}_1\theta. \tag{1.5}$$

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