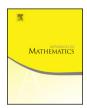


Contents lists available at ScienceDirect

Advances in Mathematics





Parking spaces [☆]



Drew Armstrong^a, Victor Reiner^{b,*}, Brendon Rhoades^c

- ^a Dept. of Mathematics, University of Miami, Coral Gables, FL 33146, United States
- ^b School of Mathematics, University of Minnesota, Minneapolis, MN 55455, United States
- ^c Dept. of Mathematics, University of California, San Diego, La Jolla, CA 92093, United States

ARTICLE INFO

Article history: Received 20 October 2012 Accepted 21 October 2014 Available online 12 November 2014 Communicated by Ezra Miller

Keywords:
Parking function
Coxeter group
Reflection group
Noncrossing
Nonnesting
Catalan
Kirkman
Narayana
Cyclic sieving
Absolute order
Rational Cherednik algebra

ABSTRACT

Let W be a Weyl group with root lattice Q and Coxeter number h. The elements of the finite torus Q/(h+1)Q are called the W-parking functions, and we call the permutation representation of W on the set of W-parking functions the (standard) W-parking space. Parking spaces have interesting connections to enumerative combinatorics, diagonal harmonics, and rational Cherednik algebras. In this paper we define two new W-parking spaces, called the noncrossing parking space and the algebraic parking space, with the following features:

- They are defined more generally for real reflection groups.
- They carry not just W-actions, but $W \times C$ -actions, where C is the cyclic subgroup of W generated by a Coxeter element.
- In the crystallographic case, both are isomorphic to the standard W-parking space.

Our Main Conjecture is that the two new parking spaces are isomorphic to each other as permutation representations of $W \times C$. This conjecture ties together several threads in the Catalan combinatorics of finite reflection groups. Even the

^{*} The first author is partially supported by NSF grant DMS-1001825. The second author is partially supported by NSF grant DMS-1001933. The third author is partially supported by NSF grant DMS-1068861.

^{*} Corresponding author.

E-mail addresses: d.armstrong@math.miami.edu (D. Armstrong), reiner@math.umn.edu (V. Reiner), bprhoades@math.ucsd.edu (B. Rhoades).

weakest form of the Main Conjecture has interesting combinatorial consequences, and this weak form is proven in all types except E_7 and E_8 . We provide evidence for the stronger forms of the conjecture, including proofs in some cases, and suggest further directions for the theory.

© 2014 Elsevier Inc. All rights reserved.

Contents

1.	Introduction	648
	1.1. Classical parking functions and spaces	
	1.2. Weyl group and real reflection group parking spaces	649
2.	Definitions and Main Conjecture	651
	2.1. Noncrossing partitions for W	651
	2.2. Nonnesting partitions for W	653
	2.3. Noncrossing and nonnesting parking functions	654
	2.4. The coincidence of W -representations	655
	2.5. The algebraic W -parking space	656
	2.6. The Main Conjecture	
3.	Consequences of the Main Conjecture	662
	3.1. First consequence: the W-action gives $Park_W^{NN} \cong_W Park_W^{NC}$	662
	3.2. Second consequence: the C-action is a cyclic sieving phenomenon	663
	3.3. Third consequence: Kirkman and Narayana numbers for W	663
4.	Proof of Proposition 2.11	665
5.	Proof of Proposition 2.13	668
	5.1. Proof of Proposition 2.13(i)	668
	5.2. Proof of Proposition 2.13(iii)	668
	5.3. Some noncrossing geometry	671
	5.4. Proof of Proposition 2.13(ii)	675
6.	Type B/C	676
	6.1. Visualizing type B/C	
	6.2. Proof of Main Conjecture (intermediate version) in type B/C	679
7.	Type D	681
	7.1. Visualizing type <i>D</i>	681
	7.2. Proof of Main Conjecture (intermediate version) in type D	683
8.	Proof of Main Conjecture (weak version) in type A	686
9.	Narayana and Kirkman polynomials	690
	9.1. Proof of Corollary 3.3	690
	9.2. Formulas for Kirkman and q-Kirkman numbers	691
10.	Inspiration: nonnesting parking functions label Shi regions	695
11.	Open problems	697
	11.1. Two basic problems	697
	11.2. Nilpotent orbits, q-Kreweras and q-Narayana numbers	698
	11.3. The Fuss parameter	698
	11.4. The near boundary cases of Kirkman numbers	700
Ackn	owledgments	701
Арре	ndix A. Etingof's proof of reducedness for a Certain hsop	701
Refer	ences	705

1. Introduction

Let W be a finite Coxeter group (finite real reflection group). (We refer to the standard references [10,30].) The main goal of this paper is to define two new objects that deserve

Download English Version:

https://daneshyari.com/en/article/4665611

Download Persian Version:

https://daneshyari.com/article/4665611

<u>Daneshyari.com</u>