Advances in Mathematics 269 (2015) 647–706

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Parking spaces ☆

Drew Armstrong [a], Victor Reiner [b,*], Brendon Rhoades [c]

[a] *Dept. of Mathematics, University of Miami, Coral Gables, FL 33146, United States*
[b] *School of Mathematics, University of Minnesota, Minneapolis, MN 55455, United States*
[c] *Dept. of Mathematics, University of California, San Diego, La Jolla, CA 92093, United States*

A R T I C L E I N F O

Article history:
Received 20 October 2012
Accepted 21 October 2014
Available online 12 November 2014
Communicated by Ezra Miller

Keywords:
Parking function
Coxeter group
Reflection group
Noncrossing
Nonnesting
Catalan
Kirkman
Narayana
Cyclic sieving
Absolute order
Rational Cherednik algebra

A B S T R A C T

Let W be a Weyl group with root lattice Q and Coxeter number h. The elements of the finite torus $Q/(h+1)Q$ are called the W-**parking functions**, and we call the permutation representation of W on the set of W-parking functions the (standard) W-**parking space**. Parking spaces have interesting connections to enumerative combinatorics, diagonal harmonics, and rational Cherednik algebras. In this paper we define two new W-parking spaces, called the **noncrossing parking space** and the **algebraic parking space**, with the following features:

- They are defined more generally for real reflection groups.
- They carry not just W-actions, but $W \times C$-actions, where C is the cyclic subgroup of W generated by a Coxeter element.
- In the crystallographic case, both are isomorphic to the standard W-parking space.

Our Main Conjecture is that the two new parking spaces are isomorphic to each other as permutation representations of $W \times C$. This conjecture ties together several threads in the Catalan combinatorics of finite reflection groups. Even the

☆ The first author is partially supported by NSF grant DMS-1001825. The second author is partially supported by NSF grant DMS-1001933. The third author is partially supported by NSF grant DMS-1068861.
* Corresponding author.
E-mail addresses: d.armstrong@math.miami.edu (D. Armstrong), reiner@math.umn.edu (V. Reiner), bprhoades@math.ucsd.edu (B. Rhoades).

http://dx.doi.org/10.1016/j.aim.2014.10.012

weakest form of the Main Conjecture has interesting combinatorial consequences, and this weak form is proven in all types except E_7 and E_8. We provide evidence for the stronger forms of the conjecture, including proofs in some cases, and suggest further directions for the theory.

Contents

1. Introduction

Let W be a finite Coxeter group (finite real reflection group). (We refer to the standard references [10,30].) The main goal of this paper is to define two new objects that deserve