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Iteration of holomorphic maps on Lie balls

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We investigate iterations of fixed-point free holomorphic self-maps on a Lie ball of any dimension, where a Lie ball is a bounded symmetric domain and the open unit ball of a spin factor which can be infinite dimensional. We describe the invariant domains of a holomorphic self-map f on a Lie ball D when f is fixed-point free and compact, and show that each limit function of the iterates (f^n) has values in a one-dimensional disc on the boundary of D. We show that the Möbius transformation g_a induced by a nonzero element a in D may fail the Denjoy–Wolff-type theorem, even in finite dimension. We determine those which satisfy the theorem.

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1. Introduction

Since the seminal works of Julia [17], Denjoy [10] and Wolff [28,29], there has been extensive literature on iteration of holomorphic maps on various complex domains in finite or infinite dimensions. In particular, iteration of holomorphic maps on the Euclidean balls and infinite dimensional Hilbert balls has been widely studied (see for example,

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[7,8,13,15,18,23,25]). These domains are bounded symmetric domains of rank one. Finite dimensional bounded symmetric domains have been classified by É. Cartan [5]. Among the *irreducible* bounded symmetric domains of rank two, Lie balls stand out as one of the four classical series of Cartan domains and are an important mathematical model for physics [12]. This motivates our study of holomorphic iteration on a Lie ball.

To investigate iteration of a holomorphic map $f : D \longrightarrow D$ on a bounded domain D in a complex Banach space, one studies the asymptotic behaviour of the iterates (f^n). For this, the invariant domains of f play a useful role and are by themselves interesting objects of study. These are defined to be the domains S in D satisfying $f(S) \subset \overline{S}$, where $\overline{S}$ denotes the closure of S. While one can establish the existence of invariant domains via a limit of hyperbolic balls, and observe that a limit function of (f^n) takes values in certain subset of the closure $\overline{D}$, the problem is to describe these domains and limit sets explicitly in such a way that definitive conclusions can be drawn. In the case of a Hilbert ball or some other domains, the latter task has been accomplished and definitive results, for example, an analogue of the Denjoy–Wolff theorem, are established. Our objective in this paper is to perform similar tasks for Lie balls. We give in Theorem 4.4, Theorem 4.6 and Section 6 an explicit description of these invariant domains, called *horospheres*, and the limit sets for a fixed-point free compact holomorphic self-map f on a Lie ball D, thereby generalising Wolff's theorem in [29] for the open unit disc in $\mathbb{C}$. Indeed, these horospheres are affinely homeomorphic to D, but we show in Proposition 6.8 that the intersection of their closures is either a point or a one-dimensional disc in the boundary of D, in contrast to the case of Hilbert balls where the intersection is always a single boundary point. Consequently, we prove in Theorem 6.17 that every limit function of the iterates (f^n) is either constant with value in the boundary of D or takes values in a one-dimensional disc of the boundary. We show for instance, that not all Möbius transformations on a Lie ball D satisfy the Denjoy–Wolff-type theorem and determine, in Theorem 5.1, which ones do. Actually, the limit function of the iterates of a Möbius transformation can take values at *every* point of a one-dimensional disc in the boundary of D. More generally, given a fixed-point free holomorphic self-map f on D with one convergent orbit, we show in Proposition 6.2 that either all limit functions of (f^n) are the same constant function with value in the boundary of D or the image of each limit function is contained in a unique one-dimensional disc on the boundary. Besides Möbius transformations, we give some examples on the three-dimensional Lie ball.

We make essential use of the Jordan algebraic structures of a spin factor to derive iteration results for a Lie ball. Every other *infinite dimensional* irreducible finite-rank bounded symmetric domain is realisable as the open unit ball of the space $L(\mathbb{C}^p, H)$ of linear operators from $\mathbb{C}^p$ to an infinite dimensional Hilbert space H. The Jordan structure of $L(\mathbb{C}^p, H)$ is markedly different and one needs different techniques to treat iteration for these domains.

Let Z be a Banach space. The closure of a set E in Z is always denoted by $\overline{E}$. Let U be the open unit ball of Z. We denote by

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