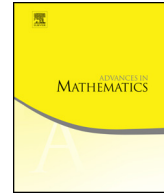




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Dyadic harmonic analysis beyond doubling measures[☆]



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ABSTRACT

We characterize the Borel measures μ on $\mathbb{R}$ for which the associated dyadic Hilbert transform, or its adjoint, is of weak-type $(1,1)$ and/or strong-type (p,p) with respect to μ . Surprisingly, the class of such measures is strictly bigger than the traditional class of dyadically doubling measures and strictly smaller than the whole Borel class. In higher dimensions, we provide a complete characterization of the weak-type $(1,1)$ for arbitrary Haar shift operators, cancellative or not, written in terms of two generalized Haar systems and these include the dyadic paraproducts. Our main tool is a new Calderón–Zygmund decomposition valid for arbitrary Borel measures which is of independent interest.

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1. Introduction

Dyadic techniques are nowadays fundamental in harmonic analysis. Their origin goes back to Hardy, Littlewood, Paley and Walsh, among others. In the context of martingale inequalities, the dyadic maximal and square functions arise as particular cases of Doob's maximal function and Burkholder's square function for martingales associated to a dyadic filtration. Similarly, singular integral operators have been traditionally modeled by martingale transforms or martingale paraproducts. These last operators can be written in terms of martingale differences and conditional expectations, so that the full strength of probability methods applies in the analysis of their boundedness properties. In the Euclidean setting, dyadic martingale differences decompose as a sum of Haar functions and therefore we can obtain expansions using the classical Haar system.

In the last years dyadic operators have attracted a lot of attention related to the so-called A_2 -conjecture. This seeks to establish that some operators obey an $L^2(w)$ estimate for every $w \in A_2$ with a constant that grows linearly in the A_2 -characteristic of w . For the maximal function this was proved by Buckley [2]. In [27], Wittwer proved the A_2 -conjecture for Haar multipliers in one dimension. The Beurling–Ahlfors transform, the Hilbert transform and the Riesz transforms were then considered by Petermichl and Volberg in [24,22,23] (see also [11]) and the A_2 -conjecture for them was shown via the representation of these operators as averages of Haar multipliers and certain dyadic operators called Haar shifts. Paraproducts were treated in [1], and with a different approach in [7]. The final solution to the A_2 -conjecture for general Calderón–Zygmund operators was obtained by Hytönen in his celebrated paper [15]. Again, a key ingredient in the proof is that Calderón–Zygmund operators can be written as averages of dyadic operators including Haar shift operators, dyadic paraproducts and their adjoints.

The *dyadic Hilbert transform* is given by

$$H_{\mathcal{D}}f(x) = \sum_{I \in \mathcal{D}} \langle f, h_I \rangle (h_{I_-}(x) - h_{I_+}(x)).$$

Here $\mathcal{D}$ denotes some dyadic grid in $\mathbb{R}$ and h_I is the Haar function associated with $I \in \mathcal{D}$: $h_I = |I|^{-1/2}(1_{I_-} - 1_{I_+})$ where I_- and I_+ are the left and right dyadic children of I . The importance of this operator comes from the fact that the classical Hilbert transform can be obtained via averaging $H_{\mathcal{D}}$ over dyadic grids, this was shown by Petermichl [21]. That $H_{\mathcal{D}}$ is bounded on $L^2(\mathbb{R})$ follows easily from the orthogonality of the Haar system. Using the standard Calderón–Zygmund decomposition one can easily obtain (see for instance [7]) that $H_{\mathcal{D}}$ is of weak-type $(1, 1)$ and therefore bounded on $L^p(\mathbb{R})$ for $1 < p < 2$. The bounds for $p > 2$ can be derived by duality and interpolation from the weak-type $(1, 1)$ of the adjoint operator.

Let us consider a Borel measure μ in $\mathbb{R}$. One can define a Haar system in a similar manner which is now orthonormal in $L^2(\mu)$. Hence, we may consider a dyadic Hilbert transform which we momentarily denote by $H_{\mathcal{D}}^{\mu}$ and ask about its boundedness properties. The boundedness on $L^2(\mu)$ is again automatic by orthogonality. The standard

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