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Stability estimates for the Radon transform with restricted data and applications

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A B S T R A C T

In this article, we prove a stability estimate going from the Radon transform of a function with limited angle-distance data to the L^p norm of the function itself, under some conditions on the support of the function. We apply this theorem to obtain stability estimates for an inverse boundary value problem with partial data.

Contents

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1. Introduction

In this work we prove a stability estimate from the Radon transform with limited angle-distance data to a local L^p-norm of the function. Our original motivation to study this problem was to obtain stability estimates for the inverse problem in electric impedance tomography (E.I.T.) proposed by Calderón. Nevertheless, we think that the results obtained on the Radon transform restricted to some partial data sets are interesting by themselves and are the main contribution of this work.

Calderón's inverse problem deals with the recovery of a conductivity γ in the interior of a smooth domain Ω from boundary measurements realized by the Dirichlet-to-Neumann map. Let u be the solution of the Dirichlet boundary value problem

$$\begin{cases} \operatorname{div}(\gamma\nabla u) = 0 & \text{in } \Omega \\ u|_{\partial\Omega} = f \in H^{\frac{1}{2}}(\partial\Omega) \end{cases} \tag{1.1}$$

where γ is a positive function of class $\mathcal{C}^2$ on $\bar{\Omega}$. The Dirichlet-to-Neumann map assigns to a function $f \in H^{\frac{1}{2}}(\partial\Omega)$ on the boundary the corresponding Neumann data of (1.1)

$$\Lambda_\gamma f = \gamma\partial_\nu u|_{\partial\Omega}$$

where ∂_ν denotes the exterior normal derivative of u. This is a bounded operator $\Lambda_\gamma : H^{\frac{1}{2}}(\partial\Omega) \to H^{-\frac{1}{2}}(\partial\Omega)$ — in fact a pseudodifferential operator of order 1 when γ is smooth. The inverse problem formulated by Calderón [12] is whether it is possible to determine γ from Λ_γ. In fact in its initial formulation, the problem concerns only positive measurable conductivities bounded from above, and it was solved in dimension 2 in this degree of generality by Astala and Päivärinta [4] and remains so far open in higher dimensions.

This question is related to the inverse problem of determining a bounded potential $q \in L^\infty(\Omega)$ in the Schrödinger equation

$$\begin{cases} -\Delta u + qu = 0 & \text{in } \Omega \\ u|_{\partial\Omega} = f \in H^{\frac{1}{2}}(\partial\Omega), \end{cases} \tag{1.2}$$

from boundary measurements. This reduction was exploited by Sylvester and Uhlmann in [45] and in combination with the boundary determination results on the conductivity obtained by Kohn and Vogelius [35] allowed them to solve the Calderón problem for smooth conductivities in dimension $n \geq 3$. When 0 is not a Dirichlet eigenvalue of the Schrödinger operator $-\Delta + q$, the measurements are implemented by the Dirichlet-to-Neumann map, which can be similarly defined as for the conductivity equation by

$$\Lambda_q f = \partial_\nu u|_{\partial\Omega}.$$

With a slight abuse of notations, we use the convention that whenever the subscript contains the letter q, the notation refers to the Dirichlet-to-Neumann map related to the