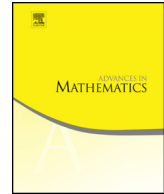




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Stability of hyperbolic manifolds with cusps under Ricci flow



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ABSTRACT

We show that every finite volume hyperbolic manifold of dimension greater than or equal to 3 is stable under rescaled Ricci flow, i.e. that every small perturbation of the hyperbolic metric flows back to the hyperbolic metric again. Note that we do not need to make any decay assumptions on this perturbation.

It will turn out that the main difficulty in the proof comes from a weak stability of the cusps which has to do with infinitesimal cusp deformations. We will overcome this weak stability by using a new analytical method developed by Koch and Lamm.

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Contents

1.	Introduction and statement of the result	413
2.	Preliminaries	415
2.1.	Hyperbolic manifolds	415
2.2.	(Modified) Ricci deTurck flow	416
2.3.	The Einstein operator	419
2.4.	A result from harmonic analysis	420
2.5.	Derivative bounds for linear and nonlinear parabolic equations	422

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2.6.	Short-time existence	426
3.	Outline of the proof	426
4.	Modified Ricci deTurck flow on a cusp	427
4.1.	Introduction	427
4.2.	The invariant and the oscillatory component of the flow	428
4.3.	The invariant component	429
4.4.	The heat kernel on the cusp	429
4.5.	Representing h_t	433
4.6.	Final argument	434
5.	Invariant modified Ricci deTurck flow on the cusp	437
5.1.	Calculations	437
5.2.	Introduction to the analytical problem	444
5.3.	The heat kernel	445
5.4.	Representing u and v using the heat kernel	447
5.5.	Estimating u^{**} and v^{**}	449
5.6.	The L^p_μ -norm	450
5.7.	Introduction of the norms	450
5.8.	The estimates	451
6.	Ricci flow on the whole manifold	461
6.1.	The heat kernel estimate	461
6.2.	The final argument	462
References	467

1. Introduction and statement of the result

In this paper, we will prove the following theorem

Theorem 1.1. *For any complete hyperbolic manifold (M^n, \bar{g}) of finite volume and dimension $n \geq 3$, there is an $\varepsilon > 0$ such that the following holds:*

If g_0 is another smooth metric on M with

$$(1 - \varepsilon)\bar{g} \leq g_0 \leq (1 + \varepsilon)\bar{g},$$

then there is a solution $(g_t)_{t \in [0, \infty)}$ to the rescaled Ricci flow equation

$$\dot{g}_t = -2\text{Ric}_{g_t} - 2(n - 1)g_t$$

starting from g_0 which exists for all time and as $t \rightarrow \infty$ we have convergence $g_t \rightarrow \bar{g}$ in the pointed smooth Cheeger–Gromov sense, i.e. there is a family of diffeomorphisms Ψ_t of M such that $\Psi_t^ g_t \rightarrow \bar{g}$ in the smooth sense on every compact subset of M .*

Moreover, ε can be chosen so that it only depends on an upper volume bound on M for $n \geq 4$ resp. an upper diameter bound on the compact part M_{cpt} of M for $n = 3$ (see Section 2.1 for more details).

Observe, that the theorem is already known in the *compact* case (see e.g. [26]). The *finite volume* case is more general than the compact case since it allows the manifold to have cusps, and hence to be noncompact (for a geometric description of these manifolds see Section 2.1). A similar stability result also holds in dimension 2. However, one has to take into account a finite dimensional deformation space of the hyperbolic structure

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