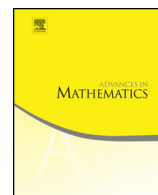




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Unitary equivalence of automorphisms of separable C^* -algebras [☆]



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ABSTRACT

We prove that the automorphisms of any separable C^* -algebra that does not have continuous trace are not classifiable by countable structures up to unitary equivalence. This implies a dichotomy for the Borel complexity of the relation of unitary equivalence of automorphisms of a separable unital C^* -algebra: Such relation is either smooth or not even classifiable by countable structures.

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1. Introduction

If A is a separable C^* -algebra, the group $\text{Aut}(A)$ of automorphisms of A is a Polish group with respect to the topology of pointwise norm convergence. An automorphism of A is called (multiplier) *inner* if it is induced by the action by conjugation of a unitary element of the multiplier algebra $M(A)$ of A . Inner automorphisms form a Borel normal subgroup $\text{Inn}(A)$ of the group of automorphisms of A . The relation of *unitary equivalence* of automorphisms of A is the coset equivalence relation on $\text{Aut}(A)$ determined by $\text{Inn}(A)$. (The reader can find more background on C^* -algebras in Section 2.) The main result presented here asserts that if A does not have continuous trace, then it is not possible to effectively classify the automorphisms of A up to unitary equivalence using countable structures as invariants; in particular this rules out classification by K-theoretic invariants. (The K-theoretic invariants of C^* -algebras were shown to be computable by a Borel function in [12, Theorem 3.3]. Even though [12, Theorem 3.3] does not consider the K-theory of $*$ -homomorphisms, it is not difficult to verify that the proof can be adapted to show that the computation of K-theory of $*$ -homomorphisms is given by a Borel functor. The main ingredient of the proof is the fact that one can enumerate in a Borel fashion dense sequences of projections and of unitary elements of the algebra and of all its amplifications [13, Lemma 3.13].) In the course of the proof of the main result we will show that the existence of an outer derivation on a C^* -algebra A is equivalent to a seemingly stronger statement, that we will refer to as Property AEP (see Definition 4.4), implying in particular the existence of an outer derivable automorphism of A .

The notion of effective classification can be made precise by means of Borel reductions in the framework of descriptive set theory (the monographs [21] and [15] are standard references for this subject). If E and E' are equivalence relations on standard Borel spaces X and X' , respectively, then a Borel reduction from E to F is a Borel function $f : X \rightarrow X'$ such that for every $x, y \in X$, xEy if and only if $f(x)E'f(y)$. The Borel function f witnesses an *effective classification* of the objects of X up to E , with E' -equivalence classes of objects of X' as invariants. This framework captures the vast majority of concrete classification results in mathematics. (In [13] and [12] the computation of most of the invariants in the theory of C^* -algebras is shown to be Borel.)

If E and F are, as before, equivalence relations on standard Borel spaces, then E is *Borel reducible* to F if there is a Borel reduction from E to F . This can be interpreted as a notion that allows one to compare the complexity of different equivalence relations. Some distinguished equivalence relations are used as benchmarks of complexity. Among these are the relation $=_Y$ of equality for elements of a Polish space Y , and the relation $\simeq_{\mathcal{C}}$ of isomorphism within some class of countable structures \mathcal{C} . If E is an equivalence relation on a standard Borel space X , we say that:

- E is *smooth* (or the elements of X are *concretely classifiable* up to E) if E is Borel reducible to $=_Y$ for some Polish space Y ;

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