



Relative Noether inequality on fibered surfaces

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ABSTRACT

We prove effective upper bounds on the global sections of nef line bundles of small generic degree over a fibered surface over a field of any characteristic. It can be viewed as a relative version of the classical Noether inequality for surfaces. As a consequence, we give a new proof of the slope inequality for fibered surface without using any stability method. The treatment is essentially different from those of Xiao, Cornalba–Harris and Moriwaki. We also study the geography problem of surfaces in positive characteristics and show that the Severi inequality is true for surfaces of general type in positive characteristic whose Albanese map is generically finite. Moreover, the geography of surfaces with Albanese fibrations is studied.

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1. Introduction

This paper is the algebraic version of the previous work [\[24\]](#) of the authors on linear series on arithmetic surfaces.

We prove effective upper bounds on the global sections of nef line bundles of small generic degree over a fibered surface over a field of any characteristic. It can be viewed as a relative version of the classical Noether inequality for surfaces.

As a consequence, we give a new proof of the slope inequality for fibered surface without using any stability method. The treatment is essentially different from those of Xiao, Cornalba–Harris and Moriwaki. We also study the geography problem of surfaces in positive characteristics and show that the Severi inequality is true for surfaces of general type in positive characteristic whose Albanese map is generically finite. Moreover, the geography of surfaces with Albanese fibrations is studied.

We would like to point out that most results in this paper, except the slope inequality, are new in positive characteristic. Nevertheless, we will state our results in full generality, since in characteristic 0 they still hold and [Theorems 1.1 and 1.2](#) have not been stated yet in previous literatures.

1.1. Relative Noether inequality

Let k be an algebraically closed field of any characteristic. Let $f : X \rightarrow Y$ be a *surface fibration of genus g* over k . That is:

- (1) X is a smooth projective surface over k ;
- (2) Y is a smooth projective curve over k ;
- (3) f is flat and the general fiber F of f is a geometrically integral curve of arithmetic genus $g := p_a(F)$.

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