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Coupling functions for domino tilings of Aztec diamonds

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ABSTRACT

The inverse Kasteleyn matrix of a bipartite graph holds much information about the perfect matchings of the system such as local statistics which can be used to compute local and global asymptotics. In this paper, we consider three different weightings of domino tilings of the Aztec diamond and show using recurrence relations, that we can compute the inverse Kasteleyn matrix. These weights are the one-periodic weighting where the horizontal edges have one weight and the vertical edges have another weight, the q^{vol} weighting which corresponds to multiplying the product of tile weights by q if we add a 'box' to the height function and the two-periodic weighting which exhibits a flat region with defects in the center.

Contents

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1. Introduction

1.1. Terminology

Domino tilings of bounded lattice regions have been extensively researched during the last twenty years. These tilings are the same as *perfect matchings* of a bounded portion G of the dual square lattice, in the following way: a *matched edge* corresponds to a domino; the fact that the dominos do not overlap means that no two matched edges share a vertex, and the fact that the dominos cover the entire region means that each vertex in the region is covered by a matched edge. In the statistical mechanics literature, one speaks of *dimer covers* rather than *perfect matchings*, and *dimers* rather than *matched edges*.

The most well-studied example of such a model is domino tilings of the *Aztec diamond* which was introduced in [11]. Here, one tiles the region $\{(x, y)\colon\ |x| + |y| \leqslant n + 1\}$ with 2 by 1 rectangles where n is the size of the Aztec diamond. There are other examples of the theory, but they involve replacing the graph G with a different one, such as the regular square–octagon lattice (giving the so-called *diabolo tilings*) or the hexagonal mesh (giving *lozenge tilings*).

By giving each edge a multiplicative weight, we can consider *random* dimer coverings: the probability of each covering is proportional to the product of the edge weights of the dimer covering. The corresponding discrete probability space is called the *dimer model.* If the graph G is bipartite (as it shall be for the rest of this paper) then each dimer covering