Advances in Mathematics 259 (2014) 376–383

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

On operator inequalities of some relative operator entropies

Ismail Nikoufar

Department of Mathematics, Payame Noor University, P.O. Box 19395-3697 Tehran, Iran

ARTICLE INFO

Article history:
Received 3 February 2014
Accepted 19 March 2014
Available online 12 April 2014
Communicated by Charles Fefferman

MSC:
81P45
15A39
47A63
15A42
81R15

Keywords:
Perspective function
Generalized perspective function
Relative operator entropy
Tsallis relative operator entropy

ABSTRACT

In our recent paper, we introduced the notions of relative operator (α, β)-entropy and Tsallis relative operator (α, β)-entropy as a parameter extensions of relative operator entropy and Tsallis relative operator entropy. In this paper, we give upper and lower bounds of these new notions according to operator (α, β)-geometric mean introduced in Nikoufar et al. (2013) [14].

1. Introduction

The classical perspective function associated to f, defined on a convex set $\mathcal{C} \subseteq \mathbb{R}^n$, is a function of two variables on the subset

$$K := \left\{ (t, s):\ s > 0,\ \frac{t}{s} \in \mathcal{C} \right\} \subseteq \mathbb{R}^{n+1}$$

E-mail address: nikoufar@pnu.ac.ir.

http://dx.doi.org/10.1016/j.aim.2014.03.019
0001-8708/

considered by $P_f(t,s) := f(\frac{t}{s})s$ (cf. [8]). Marechal defined the generalized perspective function by $P_{f\Delta g}(x,y) := f(\frac{x}{g(y)})g(y)$ on $\mathbb{R}^{n+m}$ for functions $f : \mathbb{R}^n \to (-\infty,\infty)$ and $g : \mathbb{R}^m \to (0,\infty)$ (cf. [9,10]). This generalization of perspectivity of functions has a natural operator version.

Effros [2] considered an operator version of perspectivity for commuting operators and proved in this way that the perspective of an operator convex function is operator convex as a function of two variables. Let f and h be real valued continuous functions on the closed interval $\mathbb{I}$. By recalling that if for every continuous function f, $f(A)$ commutes with every operator commuting with A (including A itself) and by restricting to positive commuting operators, Effros defined the generalized perspective function by

$$P_{f\Delta h}(A,B) := f\left(\frac{A}{h(B)}\right)h(B).$$

We defined in [12] a fully noncommutative generalized perspective of two variables (associated to f and h) by choosing an appropriate ordering and by setting

$$P_{f\Delta h}(A,B) := h(B)^{1/2} f\big(h(B)^{-1/2} A h(B)^{-1/2}\big) h(B)^{1/2},$$

where A is a self-adjoint operator and B is a strictly positive operator on a Hilbert space $\mathcal{H}$ with spectra in the closed interval $\mathbb{I}$ containing 0. The perspective of the function f is denoted by P_f and is defined by $P_f(A,B) := B^{1/2} f(B^{-1/2}AB^{-1/2})B^{1/2}$. In this approach all references to commutativity can be removed and this contribution can surely affect quantum information theory and quantum statistical mechanics. We then proved the necessary and sufficient conditions for jointly convexity of a fully noncommutative perspective and generalized perspective function.

Generalized entropies are used as alternate measures of an informational content. In particular, they may be used to study properties of the standard entropy in more general setting.

The notion of relative operator entropy was considered on strictly positive operators in noncommutative information theory [4] as follows:

$$S(A|B) := A^{\frac{1}{2}}\big(\log A^{-\frac{1}{2}} B A^{-\frac{1}{2}}\big) A^{\frac{1}{2}}.$$

This is an extension of the operator entropy defined by Nakamura and Umegaki [11] and the relative operator entropy introduced by Umegaki [15]. More generally, the generalized relative operator entropy for strictly positive operators A, B and $q \in \mathbb{R}$ defined in [7] by setting

$$S_q(A|B) = A^{1/2}\big(A^{-1/2}BA^{-1/2}\big)^q \big(\log A^{-1/2}BA^{-1/2}\big) A^{1/2}.$$

In particular, when $q = 0$, we have $S_0(A|B) = S(A|B)$. Using the notion of generalized relative operator entropy, Furuta obtained the parametric extension of operator