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Low Mach number limit for the full compressible magnetohydrodynamic equations with general initial data

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A B S T R A C T

The low Mach number limit for the full compressible magnetohydrodynamic equations with general initial data is rigorously justified in the whole space $\mathbb{R}^3$. First, the uniform-in-Mach-number estimates of the solutions in a Sobolev space are established on a finite time interval independent of the Mach number. Then the low Mach number limit is proved by combining these uniform estimate with a theorem due to Métivier and Schochet (2001) [45] for the Euler equations that gives the local energy decay of the acoustic wave equations.

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1. Introduction

In this paper we study the low Mach number limit of local smooth solutions to the following full compressible magnetohydrodynamic (MHD) equations with general initial data in the whole space $\mathbb{R}^3$ (see [27,36,38,48]):

$$\partial_t\rho + \operatorname{div}(\rho\mathbf{u}) = 0, \tag{1.1}$$

$$\partial_t(\rho\mathbf{u}) + \operatorname{div}(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla P = \frac{1}{4\pi}(\operatorname{curl}\mathbf{H})\times\mathbf{H} + \operatorname{div}\Psi(\mathbf{u}), \tag{1.2}$$

$$\partial_t\mathbf{H} - \operatorname{curl}(\mathbf{u}\times\mathbf{H}) = -\operatorname{curl}(\nu\operatorname{curl}\mathbf{H}), \quad \operatorname{div}\mathbf{H} = 0, \tag{1.3}$$

$$\begin{aligned}\partial_t\mathcal{E} + \operatorname{div}\big(\mathbf{u}(\mathcal{E}' + P)\big) &= \frac{1}{4\pi}\operatorname{div}\big((\mathbf{u}\times\mathbf{H})\times\mathbf{H}\big)\\ &\quad + \operatorname{div}\left(\frac{\nu}{4\pi}\mathbf{H}\times(\operatorname{curl}\mathbf{H}) + \mathbf{u}\Psi(\mathbf{u}) + \kappa\nabla\theta\right).\end{aligned} \tag{1.4}$$

Here the unknowns ρ, $\mathbf{u} = (u_1, u_2, u_3) \in \mathbb{R}^3$, $\mathbf{H} = (H_1, H_2, H_3) \in \mathbb{R}^3$, and θ denote the density, velocity, magnetic field, and temperature, respectively; $\Psi(\mathbf{u})$ is the viscous stress tensor given by

$$\Psi(\mathbf{u}) = 2\mu\mathbb{D}(\mathbf{u}) + \lambda\operatorname{div}\mathbf{u}\mathbf{I}_3$$

with $\mathbb{D}(\mathbf{u}) = (\nabla\mathbf{u} + \nabla\mathbf{u}^\top)/2$, $\mathbf{I}_3$ the 3×3 identity matrix, and $\nabla\mathbf{u}^\top$ the transpose of the matrix $\nabla\mathbf{u}$; $\mathcal{E}$ is the total energy given by $\mathcal{E} = \mathcal{E}' + |\mathbf{H}|^2/(8\pi)$ and $\mathcal{E}' = \rho(e + |\mathbf{u}|^2/2)$ with e being the internal energy, $\rho|\mathbf{u}|^2/2$ the kinetic energy, and $|\mathbf{H}|^2/(8\pi)$ the magnetic energy. The viscosity coefficients λ and μ of the flow satisfy $\mu > 0$ and $2\mu + 3\lambda > 0$. The parameter $\nu > 0$ is the magnetic diffusion coefficient of the magnetic field and $\kappa > 0$ the heat conductivity. For simplicity, we assume that μ, λ, ν and κ are constants. The equations of state $P = P(\rho, \theta)$ and $e = e(\rho, \theta)$ relate the pressure P and the internal energy e to the density ρ and the temperature θ of the flow.

Multiplying (1.2) by $\mathbf{u}$ and (1.3) by $\mathbf{H}/(4\pi)$ and summing over, one finds that

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\rho|\mathbf{u}|^2 + \frac{1}{8\pi}|\mathbf{H}|^2\right) + \frac{1}{2}\operatorname{div}\big(\rho|\mathbf{u}|^2\mathbf{u}\big) + \nabla P\cdot\mathbf{u}\\ &\quad = \operatorname{div}\Psi\cdot\mathbf{u} + \frac{1}{4\pi}(\operatorname{curl}\mathbf{H})\times\mathbf{H}\cdot\mathbf{u} + \frac{1}{4\pi}\operatorname{curl}(\mathbf{u}\times\mathbf{H})\cdot\mathbf{H}\\ &\qquad - \frac{\nu}{4\pi}\operatorname{curl}(\operatorname{curl}\mathbf{H})\cdot\mathbf{H}.\end{aligned} \tag{1.5}$$

Due to the identities

$$\begin{aligned}&\operatorname{div}\big(\mathbf{H}\times(\operatorname{curl}\mathbf{H})\big) = |\operatorname{curl}\mathbf{H}|^2 - \operatorname{curl}(\operatorname{curl}\mathbf{H})\cdot\mathbf{H},\\ &\operatorname{div}\big((\mathbf{u}\times\mathbf{H})\times\mathbf{H}\big) = (\operatorname{curl}\mathbf{H})\times\mathbf{H}\cdot\mathbf{u} + \operatorname{curl}(\mathbf{u}\times\mathbf{H})\cdot\mathbf{H},\end{aligned} \tag{1.6}$$