

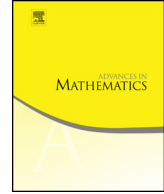


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Functional integral approach to semi-relativistic Pauli–Fierz models



Fumio Hiroshima

Faculty of Mathematics, Kyushu University, Motoooka 744, Nishiku, Fukuoka,
819-0395, Japan

ARTICLE INFO

Article history:

Received 18 June 2013

Accepted 13 February 2014

Available online 3 April 2014

Communicated by Charles Fefferman

Dedicated to Professor Asao Arai on
the occasion of his 60th birthday

Keywords:

Pauli–Fierz model

Feynman–Kac formula

Ground states

Self-adjointness

Spatial exponential decay

Gaussian dominations

Gibbs measures

ABSTRACT

By means of functional integrations spectral properties of semi-relativistic Pauli–Fierz Hamiltonians

$$H = \sqrt{(\mathbf{p} - \alpha \mathbf{A})^2 + m^2} - m + V + H_{\text{rad}}$$

in quantum electrodynamics are considered. Here \mathbf{p} is the momentum operator, \mathbf{A} a quantized radiation field on which an ultraviolet cutoff is imposed, V an external potential, H_{rad} the free field Hamiltonian and $m \geq 0$ describes the mass of electron. Two self-adjoint extensions of a semi-relativistic Pauli–Fierz Hamiltonian are defined. The Feynman–Kac type formula of e^{-tH} is given. A self-adjointness, a spatial decay of bound states, a Gaussian domination of the ground state and the existence of a measure associated with the ground state are shown. All the results are independent of values of coupling constant α , and it is emphasized that $m = 0$ is included.

© 2014 Elsevier Inc. All rights reserved.

E-mail address: hiroshima@math.kyushu-u.ac.jp.

<http://dx.doi.org/10.1016/j.aim.2014.02.015>

0001-8708/© 2014 Elsevier Inc. All rights reserved.

1. Introduction

1.1. Preliminary

In the past decade a great deal of work has been devoted to studying spectral properties of non-relativistic quantum electrodynamics in the purely mathematical point of view. In this paper we are concerned with the semi-relativistic Pauli–Fierz model (it is abbreviated as SRPF model) in quantum electrodynamics and its spectral properties by using functional integrations. The SRPF model describes a minimal interaction between semi-relativistic electrons and a massless quantized radiation field A on which an ultra-violet cutoff function is imposed. We assume throughout this paper that the electron is spinless and moves in d (≥ 3) dimensional Euclidean space for simplicity. In the case where the electron has spin $1/2$, the procedure is similar and we shall publish details somewhere. A Hamiltonian of semi-relativistic as well as non-relativistic quantum electrodynamics is usually described as a self-adjoint operator in the tensor product of a Hilbert space and a boson Fock space. In this paper instead of the boson Fock space we can formulate the Hamiltonian as a self-adjoint operator in the known Schrödinger representation in a functional realization of the boson Fock space as a space of square integrable functions with respect to the corresponding Gaussian measure. Through the Schrödinger representation a Feynman–Kac type formula of the strongly continuous one parameter semigroup generated by the SRPF Hamiltonian is given. A functional integral or a path measure approach is proven to be useful to study properties of bound states associated with embedded eigenvalues in the continuous spectrum. See e.g., [22, Sections 6 and 7]. We are interested in investigating properties of bound states and ground states of the SRPF Hamiltonian by functional integrations.

1.2. Self-adjoint extensions and functional integrations

The SRPF Hamiltonian can be realized as a self-adjoint operator bounded from below in the tensor product of $L^2(\mathbb{R}^d)$ and a boson Fock space \mathcal{F} , where $L^2(\mathbb{R}^d)$ denotes the state space of a semi-relativistic electron and \mathcal{F} that of photons. Then the decoupled Hamiltonian is given by

$$(\sqrt{\mathbf{p}^2 + m^2} - m + V) \otimes \mathbb{1} + \mathbb{1} \otimes \mathbf{H}_{\text{rad}}, \quad (1.1)$$

where $\mathbf{p} = (p_1, \dots, p_d) = (-i\partial_{x_1}, \dots, -i\partial_{x_d})$ denotes the momentum operator, m electron mass, $V : \mathbb{R}^d \rightarrow \mathbb{R}$ an external potential, and \mathbf{H}_{rad} the free field Hamiltonian on \mathcal{F} . The SRPF Hamiltonian is defined by introducing the minimal coupling by the quantized radiation field A with cutoff function $\hat{\varphi}$, i.e., replacing $\mathbf{p} \otimes \mathbb{1}$ with $\mathbf{p} \otimes \mathbb{1} - \alpha A$ and, then

$$\mathbf{H} = \sqrt{(\mathbf{p} \otimes \mathbb{1} - \alpha A)^2 + m^2} - m + V \otimes \mathbb{1} + \mathbb{1} \otimes \mathbf{H}_{\text{rad}}, \quad (1.2)$$

Download English Version:

<https://daneshyari.com/en/article/4665768>

Download Persian Version:

<https://daneshyari.com/article/4665768>

[Daneshyari.com](https://daneshyari.com)