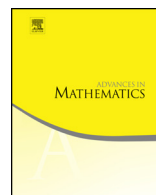




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Abelian varieties in Brill–Noether loci [☆]Ciro Ciliberto ^a, Margarida Mendes Lopes ^{b,*}, Rita Pardini ^c^a *Dipartimento di Matematica, II Università di Roma, Italy*^b *Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal*^c *Dipartimento di Matematica, Università di Pisa, Largo B. Pontecorvo, 5, 56127 Pisa, Italy*

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ABSTRACT

In this paper, improving on results of Abramovich, Harris, Debarre and Fahlaoui [1,8], we give the full classification of curves C of genus g such that a Brill–Noether locus $W_d^s(C)$, strictly contained in the jacobian $J(C)$ of C , contains a variety Z stable under translations by the elements of a positive dimensional abelian subvariety $A \subsetneq J(C)$ and such that $\dim(Z) = d - \dim(A) - 2s$, i.e., the maximum possible for such a Z .

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1. Introduction

In [1] the authors posed the problem of studying, and possibly classifying, situations like this:

- (*) C is a smooth, projective, complex curve of genus g , Z is an irreducible r -dimensional subvariety of a Brill–Noether locus $W_d^s(C) \subsetneq J^d(C)$, and Z is stable under translations by the elements of an abelian subvariety $A \subsetneq J(C)$ of dimension $a > 0$ (if so, we will say that Z is A -stable).

Actually in [1] the variety Z is the translate of a positive dimensional proper abelian subvariety of $J(C)$, while the above slightly more general formulation was given in [8].

The motivation for studying (*) resides, among other things, in a theorem of Faltings (see [9]) to the effect that if X is an abelian variety defined over a number field \mathbb{K} , and $Z \subsetneq X$ is a subvariety not containing any translate of a positive dimensional abelian subvariety of X , then the number of rational points of Z over \mathbb{K} is finite. The idea in [1] was to apply Faltings' theorem to the d -fold symmetric product $C(d)$ of a curve C defined over a number field \mathbb{K} . If C has no positive dimensional linear series of degree d , then $C(d)$ is isomorphic to its Abel–Jacobi image $W_d(C)$ in $J^d(C)$. Thus $C(d)$ has finitely many rational points over \mathbb{K} if $W_d(C)$ does not contain any translate of a positive dimensional abelian subvariety of $J(C)$. The suggestion in [1] is that, if, by contrast, $W_d(C)$ contains the translate of a positive dimensional abelian subvariety of $J(C)$, then C should be *quite special*, e.g., it should admit a map to a curve of lower positive genus (curves of this kind clearly are in situation (*)). This idea was tested in [1], where a number of partial results were proven for low values of d .

The problem was taken up in [8], see also [7], where, among other things, it is proven that if (*) holds, then $r + a + 2s \leq d$, and, if in addition $d + r \leq g - 1$, then $r + a + 2s = d$ if and only if:

- (a) there is a degree 2 morphism $\varphi: C \rightarrow C'$, with C' a smooth curve of genus a , such that $A = \varphi^*(J(C'))$ and $Z = W_{d-2a-2s}(C) + \varphi^*(J^{a+s}(C'))$.

In [8] there is also the following example with $(d, s) = (g - 1, 0)$:

- (b) there is an (étale) degree 2 morphism $\varphi: C \rightarrow C'$, with C' a smooth curve of genus $g' = r + 1$, A is the Prym variety of φ and $Z \subset W_{g-1}(C)$ is the connected component of $\varphi_*^{-1}(K_{C'})$ consisting of divisor classes D with $h^0(\mathcal{O}_C(D))$ odd, where $\varphi_*: J^{g-1}(C) \rightarrow J^{g-1}(C')$ is the *norm map*. One has $Z \cong A$, hence $r = a$.

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