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A characterization of varieties whose universal cover is a bounded symmetric domain without ball factors ☆

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ABSTRACT

We give two characterizations of varieties whose universal cover is a bounded symmetric domain without ball factors in terms of the existence of a holomorphic endomorphism σ of the tensor product $T \otimes T^\vee$ of the tangent bundle T with the cotangent bundle $T^\vee$. To such a curvature type tensor σ one associates the first Mok characteristic cone $\mathcal{CS}$, obtained by projecting on T the intersection of $ker(\sigma)$ with the space of rank 1 tensors. The simpler characterization requires that the projective scheme associated to $\mathcal{CS}$ be a finite union of projective varieties of given dimensions and codimensions in their linear spans which must be skew and generate.

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Bounded symmetric domains
Ample canonical bundle

1. Introduction

A central problem in the theory of complex manifolds is the one of determining the compact complex manifolds X whose universal covering $\tilde{X}$ is biholomorphic to a bounded domain $\Omega \subset \mathbb{C}^n$.

A first important restriction is given by theorems by Siegel and Kodaira [14,19] extending to several variables a result of Poincaré, and asserting that necessarily such a manifold X is projective and has ample canonical divisor K_X.

A restriction on Ω is given by another theorem of Siegel ([18], cf. also [10]) asserting that Ω must be holomorphically convex.

The question concerning which domains occur was partly answered by Borel [4] who showed that, given a bounded symmetric domain $\Omega \subset \mathbb{C}^n$, there exists a properly discontinuous group $\Gamma \subset \mathrm{Aut}(\Omega)$ which acts freely on Ω and is cocompact (i.e., is such that $X =: \Omega/\Gamma$ is a compact complex manifold with universal cover $\cong \Omega$).

We consider the following question: given a bounded domain $\Omega \subset \mathbb{C}^n$, how can we tell when a projective manifold X with ample canonical divisor K_X has Ω as universal covering?

The question was solved by Yau [21] in the case of a ball, using the theorem of Aubin and Yau (see [22,1]) asserting the existence of Kähler–Einstein metrics for varieties with ample canonical bundle. The existence of such metrics, joint to some deep knowledge of the differential geometry of bounded symmetric domains, allows to obtain more general results (see also [3,8,12] as general references).

Together with Franciosi [5] we took up the question for the case of a polydisk, and a fully satisfactory answer was found in [6] for the special case where the bounded symmetric domain has all factors of tube type, i.e., the domain is biholomorphic, via the Cayley transform, to some **tube domain**

$$\Omega = V + i\mathfrak{C},$$

where V is a real vector space and $\mathfrak{C} \subset V$ is an open self-dual cone containing no lines.

The main results in the tube case are as follows:

Theorem 1.1. *(See [6].) Let X be a compact complex manifold of dimension n with K_X ample.*

Then the following two conditions (1) *and* (1′), *resp.* (2) *and* (2′) *are equivalent:*

(1) *X admits a slope zero tensor $0 \neq \psi \in H^0(S^{mn}(\Omega^1_X)(-mK_X))$ (for some positive integer m);*

(1′) *$X \cong \Omega/\Gamma$, where Ω is a bounded symmetric domain of tube type and Γ is a cocompact discrete subgroup of* $\mathrm{Aut}(\Omega)$ *acting freely.*