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Finite traces and representations of the group of infinite matrices over a finite field

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ABSTRACT

The article is devoted to the representation theory of locally compact infinite-dimensional group $\mathbb{GLB}$ of almost upper-triangular infinite matrices over the finite field with q elements. This group was defined by S.K., A.V., and Andrei Zelevinsky in 1982 as an adequate $n=\infty$ analogue of general linear groups $\mathbb{GL}(n,q)$. It serves as an alternative to $\mathbb{GL}(\infty,q)$, whose representation theory is poor.

Our most important results are the description of semifinite unipotent traces (characters) of the group $\mathbb{GLB}$ via certain probability measures on the Borel subgroup $\mathbb{B}$ and the construction of the corresponding von Neumann factor representations of type II_∞.

As a main tool we use the subalgebra $\mathcal{A}(\mathbb{GLB})$ of smooth functions in the group algebra $L_1(\mathbb{GLB})$. This subalgebra is an inductive limit of the finite-dimensional group algebras $\mathbb{C}(\mathbb{GL}(n,q))$ under parabolic embeddings.

As in other examples of the asymptotic representation theory we discover remarkable properties of the infinite case which does not take place for finite groups, like multiplicativity of indecomposable characters or connections to probabilistic concepts.

The infinite dimensional Iwahori–Hecke algebra $\mathcal{H}_q(\infty)$ plays a special role in our considerations and allows to understand the deep analogy of the developed theory with the

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representation theory of infinite symmetric group $S(\infty)$ which had been intensively studied in numerous previous papers.

Contents

To the memory of Andrei Zelevinsky

0. Historical preface

My joint work with S. Kerov on the asymptotic representation theory of the matrix groups $\mathbb{GL}(n,q)$ over finite field as the rank n grows to infinity, was started at the beginning of 80s as a continuation of our papers devoted to analogous problems for symmetric groups of growing ranks at the end of 70-th. It is a part of what I called "the asymptotic representation theory".

The "trivial" embedding $\mathbb{GL}(n,q) \hookrightarrow \mathbb{GL}(n+1,q)$ does not lead to an interesting or useful theory. However, another "true" (i.e. parabolic) embedding of the group algebras of $\mathbb{GL}(n,q)$ was well-known starting from the very first papers on the representation theory of $\mathbb{GL}(n,q)$ (see [23,76,13], etc.). It was used by A. Zelevinsky and us (see [62]) to define