

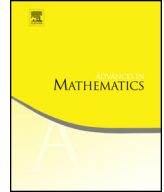


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Formal plethories

Tilman Bauer

Department of Mathematics, Kungliga Tekniska Högskolan, Lindstedtsvägen 25, 10044 Stockholm, Sweden

ARTICLE INFO

Article history:

Received 20 September 2011

Accepted 25 December 2013

Available online 16 January 2014

Communicated by Matthew Ando

MSC:

16W99

18D99

18D20

55S25

14L05

13A99

Keywords:

Plethory

Unstable cohomology operations

Two-monoidal category

Formal algebra scheme

Biring

Hopf ring

ABSTRACT

Unstable operations in a generalized cohomology theory E give rise to a functor from the category of algebras over E_* to itself which is a colimit of representable functors and a comonoid with respect to composition of such functors. In this paper I set up a framework for studying the algebra of such functors, which I call formal plethories, in the case where E_* is a Prüfer ring. I show that the “logarithmic” functors of primitives and indecomposables give linear approximations of formal plethories by bimonoids in the 2-monoidal category of bimodules over a ring.

© 2013 Elsevier Inc. All rights reserved.

Contents

1. Introduction	498
1.1. Outline of the paper	503
List of notations	503
2. Functors which are filtered colimits of representable functors	506
2.1. Two-algebra	506

E-mail address: tilmanb@kth.se.

2.2.	Ind-representable functors	509
3.	Formal bimodules	511
4.	Formal algebra and module schemes	513
4.1.	Operations in cohomology theories give formal algebra schemes	520
5.	The structure of the category of formal bimodules	522
6.	The structure of the category of formal module schemes	524
6.1.	Free/cofree adjunctions for formal module schemes and enrichments	525
6.2.	Tensor products of formal module schemes	532
7.	The structure of the category of formal algebra schemes	534
8.	Formal coalgebras and formal plethories	535
8.1.	2-Monoidal categories	535
8.2.	The 2-monoidal categories of formal algebra schemes and formal bimodules	538
9.	Primitives and indecomposables	542
9.1.	Indecomposables	542
9.2.	Primitives	545
9.3.	Primitives and indecomposables of formal algebra schemes	548
9.4.	Primitives and indecomposables of formal plethories	552
10.	Dualization	555
11.	Formal plethories and the unstable Adams spectral sequence	559
	Acknowledgments	562
	Appendix A. Pro-categories and lattices	562
	Appendix B. Pro- and ind-categories and their enrichments	564
	References	568

1. Introduction

Let k be a commutative ring. From an algebro-geometric point of view, the category of representable endofunctors of commutative k -algebras can be considered as affine schemes over k with a structure of a k -algebra on them. Composition of such representable endofunctors constitutes a non-symmetric monoidal structure \circ . A *plethory* is such a representable endofunctor F of k -algebras which is a comonoid with respect to \circ , i.e. which is equipped with natural transformations $F \rightarrow \text{id}$ and $F \rightarrow F \circ F$ such that coassociativity and counitality conditions are satisfied. The algebra of plethories was first studied by Tall andraith [36] and then extended by Borger and Wieland [10]. The aim of this paper is to extend the theory of plethories to the setting of graded *formal* schemes and to study linearizations of them. The motivation for doing this comes from topology.

Let E be a homotopy commutative ring spectrum representing a cohomology theory E^* . For any space X , $E^*(X)$ is naturally an algebra over the ring of coefficients E_* of E ; furthermore, there is an action

$$E^n(\underline{E}_m) \times E^m(X) \rightarrow E^n(X)$$

by unstable operations. Here \underline{E}_m denotes the m th space in the Ω -spectrum associated to E . The bigraded E_* -algebra $E^*(\underline{E}_*)$ almost qualifies as the representing object of a plethory, but not quite. In order for $E^*(\underline{E}_*)$ to have the required structure maps (the ring structure on the spectrum of this ring must come from a coaddition and a

Download English Version:

<https://daneshyari.com/en/article/4665818>

Download Persian Version:

<https://daneshyari.com/article/4665818>

[Daneshyari.com](https://daneshyari.com)