

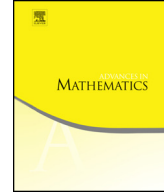


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Asymptotic expansions for the constants of Landau and Lebesgue

Chao-Ping Chen^{a,*}, Junesang Choi^{b,1}

^a School of Mathematics and Informatics, Henan Polytechnic University, Jiaozuo City 454003, Henan Province, People's Republic of China

^b Department of Mathematics, Dongguk University, Gyeongju 780-714, Republic of Korea

ARTICLE INFO

Article history:

Received 31 October 2012

Accepted 25 December 2013

Available online 17 January 2014

Communicated by Gang Tian

MSC:

primary 26D15

secondary 33B15

Keywords:

Constants of Landau and Lebesgue
Gamma function

Psi function

Asymptotic expansion

Stirling numbers of the second kind

Bernoulli numbers and polynomials

Bell polynomials

Partition function

ABSTRACT

The constants of Landau and Lebesgue are defined, for all integers $n \geq 0$, in order, by

$$G_n = \sum_{k=0}^n \frac{1}{16^k} \binom{2k}{k}^2 \quad \text{and} \quad L_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{\sin((n + \frac{1}{2})t)}{\sin(\frac{1}{2}t)} \right| dt,$$

which play important roles in the theories of complex analysis and Fourier series, respectively. Certain inequalities and asymptotic expansions for the constants G_n and L_n have been investigated by many authors. Here we aim at establishing new asymptotic expansions for the constants G_n and L_n of Landau and Lebesgue, respectively, by mainly using Bell polynomials and the partition function.

© 2014 Elsevier Inc. All rights reserved.

* Corresponding author.

E-mail addresses: chenchaoping@sohu.com (C.-P. Chen), junesang@mail.dongguk.ac.kr (J. Choi).

¹ Research is supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (2012-0002957).

1. Introduction and preliminaries

The Landau constants are defined by

$$G_n = \sum_{k=0}^n \frac{1}{16^k} \binom{2k}{k}^2 \quad (n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}, \mathbb{N} := \{1, 2, 3, \dots\}), \tag{1.1}$$

which play an important role in the theory of complex analysis. More precisely, in 1913, Landau [19] proved that if $f(z) = \sum_{k=0}^\infty a_k z^k$ is an analytic function in the unit disc $\mathcal{D} := \{z \in \mathbb{C} : |z| < 1\}$, \mathbb{C} being the set of complex numbers, which satisfies $|f(z)| < 1$ for all $z \in \mathcal{D}$, then $|\sum_{k=0}^n a_k| \leq G_n$ ($n \in \mathbb{N}_0$) whose bounds are seen to be optimal.

The Lebesgue constants are defined by

$$L_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{\sin((n + \frac{1}{2})t)}{\sin(\frac{1}{2}t)} \right| dt \quad (n \in \mathbb{N}_0), \tag{1.2}$$

which play an important role in the theory of Fourier series. More in detail, in 1906, Lebesgue [20] proved the following result: Assume a function f is integrable on the interval $[-\pi, \pi]$ and $S_n(f, x)$ is the n th partial sum of the Fourier series of f . That is,

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(kt) dt \quad (k \in \mathbb{N}_0) \quad \text{and} \quad b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(kt) dt \quad (k \in \mathbb{N})$$

and

$$S_n(f, x) = \frac{a_0}{2} + \sum_{k=1}^n (a_k \cos(kx) + b_k \sin(kx)) \quad (n \in \mathbb{N}_0),$$

where the empty sum is (as usual, throughout this paper) understood to be nil. If $|f(x)| \leq 1$ for all $x \in [-\pi, \pi]$, then

$$S_n(f, x) \leq L_n \quad (n \in \mathbb{N}_0). \tag{1.3}$$

It is noted that L_n is the smallest possible constant for which the inequality (1.3) holds true for all continuous functions f on $[-\pi, \pi]$.

Certain inequalities and asymptotic expansions for the constants G_n and L_n have been investigated by many authors (see, e.g., [5,6,10,16,23–27,33]). Here, in this paper, we aim at establishing new asymptotic expansions for the constants G_n and L_n of Landau and Lebesgue, respectively, by mainly using Bell polynomials and the partition function.

For our purpose, the following lemmas are required in the sequel.

Download English Version:

<https://daneshyari.com/en/article/4665820>

Download Persian Version:

<https://daneshyari.com/article/4665820>

[Daneshyari.com](https://daneshyari.com)