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2-blocks with abelian defect groups

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ABSTRACT

We give a classification, up to Morita equivalence, of 2-blocks of quasi-simple groups with abelian defect groups. As a consequence, we show that Donovan's conjecture holds for elementary abelian 2-groups, and that the entries of the Cartan matrices are bounded in terms of the defect for arbitrary abelian 2-groups. We also show that a block with defect groups of the form $C_{2^m} \times C_{2^m}$ for $m \geqslant 2$ has one of two Morita equivalence types and hence is Morita equivalent to the Brauer correspondent block of the normaliser of a defect group. This completes the analysis of the Morita equivalence types of 2-blocks with abelian defect groups of rank 2, from which we conclude that Donovan's conjecture holds for such 2-groups. A further application is the completion of the determination of the number of irreducible characters in a block with abelian defect groups of order 16. The proof uses the classification of finite simple groups.

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1. Introduction

Let k be an algebraically closed field of prime characteristic ℓ, and let $\mathcal{O}$ be a discrete valuation ring with residue field k. Let G be a finite group, and let B be a block of the

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group algebra $\mathcal{O}G$ with defect group D. Assume that $\mathcal{O}$ contains a primitive $|D|$-th root of unity.

The motivation for this paper is Donovan's conjecture, which states that for a fixed ℓ-group D, there should only be a finite number of Morita equivalence classes of blocks with defect groups isomorphic to D. This conjecture is stated for blocks with respect to k, but it is also expected to hold for blocks with respect to $\mathcal{O}$ (however, there are key results used in reduction arguments for the conjecture that at present are only known for k).

The main result is that every 2-block of a quasi-simple group with an abelian defect group is either one of a short list of exceptional cases or is Morita equivalent over $\mathcal{O}$ to either a block covered by a nilpotent block, or to a tensor product of a nilpotent block with a block with Klein 4-defect groups (see Theorem 6.1). Blocks covered by nilpotent blocks are treated in [33], where it is shown that they are Morita equivalent to their Brauer correspondent in the normaliser of a defect group. The Morita equivalences are achieved through the Bonnafé–Rouquier correspondence.

The above may be viewed as being in the spirit of Walter's classification of simple groups with abelian Sylow 2-subgroups, and it is hoped that it will eventually be used to tackle Donovan's conjecture for 2-blocks with arbitrary abelian defect groups. We begin here with some cases which we may tackle with tools already at our disposal.

We prove that Donovan's conjecture holds for 2-blocks with elementary abelian defect groups (Theorem 8.3). In this case we are restricted to blocks defined over k because we are reliant on the results of [25].

A conjecture of Brauer (from his Problem 22) represents a weak version of Donovan's conjecture. It states that for a given ℓ-group D, there is a bound on the entries of the Cartan matrix of a block with defect groups isomorphic to D. This conjecture has been reduced to quasi-simple groups by Düvel in [11]. We use our main result to show that the conjecture holds for all abelian 2-groups (Theorem 9.2).

We also consider the case that D is abelian of rank 2, so that D is isomorphic to a direct product $C_{2^m} \times C_{2^n}$ of two cyclic subgroups C_{2^m} and C_{2^n}. If $m \neq n$, then every block with defect group D is necessarily nilpotent. The case $m = n = 1$ is the Klein 4-case, which is already well known by [14]. We show the following:

Theorem 1.1. *Let G be a finite group and B a block of $\mathcal{O}G$ with defect group D. Suppose that $D \cong C_{2^m} \times C_{2^m}$ for some $m \geqslant 2$. Then B is Morita equivalent to either $\mathcal{O}D$ or $\mathcal{O}(D \rtimes C_3)$.*

By the remarks above, this completes the analysis of 2-blocks with abelian defect groups of rank 2, including the verification of Donovan's conjecture for these groups.

We are also able to show that Donovan's conjecture holds for groups of the form $C_{2^m} \times C_{2^m} \times C_2$ for $m \geqslant 3$ (Theorem 11.1).

Finally, we complete the determination of the number of irreducible characters and irreducible Brauer characters in a block with abelian defect groups of order 16 (Theorem 10.4). This was completed modulo one case in [27].

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