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Finite quotients of groups of I-type ☆

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ABSTRACT

To every group of I-type, we associate a finite quotient group that plays the role that Coxeter groups play for Artin–Tits groups. Since groups of I-type are examples of Garside groups, this answers a question of D. Bessis in the particular case of groups of I-type. Groups of I-type are related to finite set-theoretical solutions of the Yang–Baxter equation. So, our result provides a new tool to attack the problem of the classification of finite set-theoretical solutions of the Yang–Baxter equation.

0. Introduction

The motivation that led to develop Garside group theory at the end of the 1990s [10] or, more recently, to develop Garside family theory [9] was to extract the main ideas of Garside's theory of braids [14] and to provide a general framework for the study of other

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groups or, more generally, categories. This approach led to many developments in the last decade and most of the main objects appearing in the context of braid groups have been generalized to Garside theory framework. Braid groups are related to symmetric groups, indeed the symmetric group on n elements is a quotient of the braid group on n strands and the same relation exists between Artin–Tits groups and Coxeter groups. Tits associates to each Coxeter group a group (named Artin–Tits), such that the Coxeter group is its quotient [20]. Braid groups and Artin–Tits groups associated with finite Coxeter groups are seminal examples of Garside groups. Therefore, a natural problem, which was addressed by Bessis in [1], is to decide which Garside groups can be associated an object (a *generating generated group*) that plays the role that the symmetric group plays for the braid group (see Section 1.3). At the present time, this question remains widely open, even if partial results exist [1]. Our approach to the study of this problem is to consider a specific family of Garside groups. The first author shows that Yang–Baxter theory provides a large family of Garside groups [4]. More precisely, to each non-degenerate and symmetric set-theoretical solution of the Yang–Baxter equation is associated a group called the *structure group* [12] and this group is Garside [4]. There exists a one-to-one correspondence between structure groups and groups of I-type [17,18]. In the present paper, we associate to each structure group a finite group that plays the role that Coxeter groups play for Artin–Tits groups. Assuming the structure group satisfies an additional property, denoted by $(\mathcal{C})$, we also obtain a presentation of the finite quotient. We postpone definitions to Section 1 and state our main result:

Theorem. *(Corollary 2.10 and Propositions 3.3 and 3.8.) Let (X, S) be a non-degenerate and symmetric set-theoretical solution of the Yang–Baxter. Denote by n the cardinality of X, by $G(X, S)$ its structure group and by $M(X, S)$ its associated Garside monoid. Then*

(1) There is a finite quotient $W(X, S)$ of $G(X, S)$ that is a generating generated group for $M(X, S)$.

(2) If $M(X, S)$ satisfies Property $(\mathcal{C})$, then $W(X, S)$ is a generating generated section for $M(X, S)$. The order of $W(X, S)$ is 2^n and there is an exact sequence

$$1 \to N(X, S) \to G(X, S) \to W(X, S) \to 1 \tag{0.1}$$

where $N(X, S)$ is a free Abelian group of rank n.

Jespers and Okninski show that structure groups of non-degenerate and symmetric set-theoretical solutions of the Yang–Baxter are Abelian-by-Finite [18, Cor. 2.4]. The finite quotient groups obtained in their construction are called IYB groups [2]. The finite quotient groups we obtain here are different from the IYB groups (see Section 2) and it would be interesting to understand the connection between the classes of finite groups obtained in each construction. In [2], the authors suggested to use IYB groups in order to classify finite set-theoretical solutions of the Yang–Baxter equation, and in [3] an