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Analyticity of the total ancestor potential in singularity theory

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K. Saito's theory of primitive forms gives a natural semi-simple Frobenius manifold structure on the space of miniversal deformations of an isolated singularity. On the other hand, Givental introduced the notion of a total ancestor potential for every semi-simple point of a Frobenius manifold and conjectured that in the settings of singularity theory his definition extends analytically to non-semisimple points as well. In this paper we prove Givental's conjecture by using the Eynard–Orantin recursion.

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1. Introduction

The Gromov–Witten invariants of a compact algebraic manifold V are by definition a virtual count of holomorphic maps from a Riemann surface to V satisfying various incidents constraints. Although the rigorous definition of the Gromov–Witten invariants is very complicated, when it comes to computations, quite a bit of techniques were developed. One of the most exciting achievements is due to Givental who conjectured that under some technical conditions (which amount to saying that V has sufficiently many rational curves) we can reconstruct the higher genus invariants in terms of genus 0 and the higher genus Gromov–Witten invariants of the point. Givental's conjecture was proved recently by Teleman [22] and its impact on other areas of mathematics, such as integrable systems and the theory of quasi-modular forms is a subject of an ongoing investigation (see [5,17]).

The higher genus reconstruction formalism of Givental (see [10] or Section 3 bellow) is most naturally formulated in the abstract settings of the so-called *semi-simple Frobenius manifolds* (see [7] for some background on Frobenius manifolds). In the case of Gromov–Witten theory, the Frobenius structure is given on the vector space $H^*(V;\mathbb{C})$ and it is induced from the quantum cup product. More precisely, Givental defined the total ancestor potential of a semi-simple Frobenius manifold which in the case of Gromov–Witten theory coincides with a generating function of the so-called *ancestor* Gromov–Witten invariants (see [11]).

In this paper we study the total ancestor potential of the semi-simple Frobenius manifold arising in singularity theory. Let $f \in \mathcal{O}_{\mathbb{C}^{2l+1},0}$ be the germ of a holomorphic function with an isolated critical point at 0, i.e., the local algebra $H := \mathcal{O}_{\mathbb{C}^{2l+1},0}/(f_{x_0}, \ldots, f_{x_{2l}})$ is a finite dimensional vector space (over $\mathbb{C}$). The dimension is called *multiplicity* of the critical point and it will be denoted by N. We fix a miniversal deformation $F(t,x)$, $t \in B$ and a primitive form ω in the sense of K. Saito [19,21], so that B inherits a Frobenius structure (see [14,20]). Let B_{ss} be the set of points $t_0 \in B$, such that the critical values $u_1(t), \ldots, u_N(t)$ of $F(t,\cdot)$ form a coordinate system for t in a neighborhood of t_0. In such coordinates the product and the residue pairing assume a diagonal form which means that the corresponding Frobenius algebra is semi-simple. Let $\mathbf{t} = \{t_{k,i}\}_{k=0,1,\ldots}^{i=1,\ldots,N}$ be a sequence of formal variables. For every $t \in B_{\mathrm{ss}}$ we denote by $\mathcal{A}_t(\hbar;\mathbf{t})$ the total ancestor potential of the Frobenius structure (cf. Section 3.2). It is a formal series in $\mathbb{C}((\hbar))[[\mathbf{t}]]$, whose coefficients are analytic functions in $t \in B_{ss}$. A priori the coefficients could have poles along the caustic $B \setminus B_{\mathrm{ss}}$. Our main result is the following.