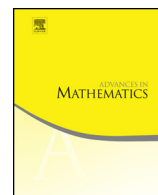




ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

[www.elsevier.com/locate/aim](http://www.elsevier.com/locate/aim)

# Logarithmic bump conditions and the two-weight boundedness of Calderón–Zygmund operators ☆

David Cruz-Uribe <sup>a</sup>, Alexander Reznikov <sup>b</sup>, Alexander Volberg <sup>b,\*</sup><sup>a</sup> Department of Mathematics, Trinity College, USA<sup>b</sup> Department of Mathematics, Michigan State University, East Lansing, MI 48824, USA

## ARTICLE INFO

*Article history:*

Received 14 January 2012

Accepted 22 January 2014

Communicated by Charles Fefferman

*MSC:*

42B20

42B35

47A30

*Keywords:*

Calderón–Zygmund operators

Carleson embedding theorem

Bellman function

Stopping time

Bump conditions

Orlicz norms

## ABSTRACT

We prove that if a pair of weights  $(u, v)$  satisfies a sharp  $A_p$ -bump condition in the scale of all log bumps or certain loglog bumps, then Haar shifts map  $L^p(v)$  into  $L^p(u)$  with a constant quadratic in the complexity of the shift. This in turn implies the two weight boundedness for all Calderón–Zygmund operators. This gives a partial answer to a long-standing conjecture. We also give a partial result for a related conjecture for weak-type inequalities. To prove our main results we combine several different approaches to these problems; in particular we use many of the ideas developed to prove the  $A_2$  conjecture. As a byproduct of our work we also disprove a conjecture by Muckenhoupt and Wheeden on weak-type inequalities for the Hilbert transform. This is closely related to the recent counterexamples of Reguera, Scurry and Thiele.

© 2014 Elsevier Inc. All rights reserved.

☆ The first author is supported by the Stewart-Dorwart faculty development fund at Trinity College and by grant MTM2009-08934 from the Spanish Ministry of Science and Innovation; the third author is supported by the NSF under the grant DMS-0758552.

\* Corresponding author.

*E-mail addresses:* [David.CruzUribe@trincoll.edu](mailto:David.CruzUribe@trincoll.edu) (D. Cruz-Uribe), [reznikov@ymail.com](mailto:reznikov@ymail.com) (A. Reznikov), [volberg@math.msu.edu](mailto:volberg@math.msu.edu) (A. Volberg).

*URL:* <http://sashavolberg.wordpress.com> (A. Volberg).

## 1. Introduction

In this paper we prove several partial results related to a pair of long-standing conjectures in the theory of two-weight norm inequalities. To state the conjectures and our results we recall a few facts about Orlicz spaces; see [4, Chapter 5] for complete details. Given a Young function  $A$ , the complementary function  $\bar{A}$  is the Young function that satisfies

$$t \leq A^{-1}(t)\bar{A}^{-1}(t) \leq 2t, \quad t > 0.$$

We will say that a Young function  $\bar{A}$  satisfies the  $B_{p'}$  condition,  $1 < p < \infty$ , if for some  $c > 0$ ,

$$\int_c^\infty \frac{\bar{A}(t)}{t^{p'}} \frac{dt}{t} < \infty.$$

If  $A$  and  $\bar{A}$  are doubling (i.e., if  $A(2t) \leq CA(t)$ , and similarly for  $\bar{A}$ ), then  $\bar{A} \in B_{p'}$  if and only if

$$\int_c^\infty \left( \frac{t^p}{A(t)} \right)^{p'-1} \frac{dt}{t} < \infty.$$

**Remark 1.** As we will see with specific examples below, if  $\bar{A} \in B_{p'}$ , then  $\bar{A}(t) \lesssim t^{p'}$  and  $A(t) \gtrsim t^p$ .

Given  $p$ ,  $1 < p < \infty$ , let  $A$  and  $B$  be Young functions such that  $\bar{A} \in B_{p'}$  and  $\bar{B} \in B_p$ . We say that the pair of weights  $(u, v)$  satisfies an  $A_p$  bump condition with respect to  $A$  and  $B$  if

$$\sup_Q \|u^{1/p}\|_{A,Q} \|v^{-1/p}\|_{B,Q} < \infty, \quad (1)$$

where the supremum is taken over all cubes  $Q$  in  $\mathbb{R}^d$ , and the Luxemburg norm is defined by

$$\|f\|_{A,Q} = \inf \left\{ \lambda > 0: \frac{1}{|Q|} \int_Q A(|f(x)|/\lambda) \, dx \leq 1 \right\}.$$

If (1) holds, then it is conjectured that

$$T : L^p(v) \rightarrow L^p(u). \quad (2)$$

Similarly, if the pair  $(u, v)$  satisfies the weaker condition

Download English Version:

<https://daneshyari.com/en/article/4665930>

Download Persian Version:

<https://daneshyari.com/article/4665930>

[Daneshyari.com](https://daneshyari.com)