Available online at www.sciencedirect.com
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Advances in Mathematics 240 (2013) 24–60
ADVANCES IN Mathematics
www.elsevier.com/locate/aim

Plane posets, special posets, and permutations

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Received 5 October 2012; accepted 5 March 2013
Available online 28 March 2013

Communicated by Michel Van den Bergh

Abstract

We study the self-dual Hopf algebra $\mathcal{H}_{\mathcal{SP}}$ of special posets introduced by Malvenuto and Reutenauer and the Hopf algebra morphism from $\mathcal{H}_{\mathcal{SP}}$ to the Hopf algebra of free quasi-symmetric functions **FQSym** given by linear extensions. In particular, we construct two Hopf subalgebras both isomorphic to **FQSym**; the first one is based on plane posets, the second one on heap-ordered forests. An explicit isomorphism between these two Hopf subalgebras is also defined, with the help of two combinatorial transformations on special posets. The restriction of the Hopf pairing of $\mathcal{H}_{\mathcal{SP}}$ to these Hopf subalgebras and others is also studied, as well as certain isometries between them. These problems are solved using duplicial and dendriform structures.
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MSC: 06A11; 05A05; 16W30; 17A30

Keywords: Special posets; Permutations; Self-dual Hopf algebras; Duplicial algebras; Dendriform algebras

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http://dx.doi.org/10.1016/j.aim.2013.03.007

0. Introduction

The Hopf algebra of double posets is introduced in [17]. Recall that a *double poset* is a finite set with two partial orders; the set of isoclasses of double posets is given a structure of monoid, with a product called *composition* (Definition 4). The algebra of this monoid is given a coassociative coproduct, with the help of the notion of *ideal* of a double poset. We then obtain a graded, connected Hopf algebra, non commutative and non cocommutative. This Hopf algebra $\mathcal{H}_{\mathcal{DP}}$ is self-dual: it has a nondegenerate Hopf pairing $\langle -, - \rangle$, such that the pairing of two double posets is given by the number of *pictures* between these double posets (Definition 6); see [6] for more details on the nondegeneracy of this pairing.

Other algebraic structures are constructed on $\mathcal{H}_{\mathcal{DP}}$ in [6]. In particular, a second product is defined on $\mathcal{H}_{\mathcal{DP}}$, making it a free 2-*As* Hopf algebra [13]. As a consequence, this object is closely related to operads and the theory of combinatorial Hopf algebras [14]. In particular, it contains the free 2-*As* algebra on one generator: this is the Hopf subalgebra $\mathcal{H}_{\mathcal{WNP}}$ of *WN posets*, see Definition 3. Another interesting Hopf subalgebra $\mathcal{H}_{\mathcal{PP}}$ is given by *plane posets*, that is to say double poset with a particular condition of (in)compatibility between the two orders (Definition 2).

We investigate in the present text the algebraic properties of the family of *special posets*, that is to say double posets such that the second order is total [17]. They generate a Hopf subalgebra of $\mathcal{H}_{\mathcal{DP}}$ denoted by $\mathcal{H}_{\mathcal{SP}}$. For example, as explained in [6], the two partial orders of a plane poset allow to define a third, total order, so plane posets can also be considered as special posets: this defines an injective morphism of Hopf algebras from $\mathcal{H}_{\mathcal{PP}}$ to $\mathcal{H}_{\mathcal{SP}}$. Its image is denoted by $\mathcal{H}_{\mathcal{SPP}}$. Another interesting Hopf subalgebra of $\mathcal{H}_{\mathcal{SP}}$ is generated by the set of *ordered forests*;

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