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Radon transformation on reductive symmetric spaces: Support theorems

Job J. Kuit

Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, 2100 Copenhagen, Denmark

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Abstract

We introduce a class of Radon transforms for reductive symmetric spaces, including the horospherical transforms, and derive support theorems for these transforms.

A reductive symmetric space is a homogeneous space G/H for a reductive Lie group G of the Harish-Chandra class, where H is an open subgroup of the fixed-point subgroup for an involution σ on G. Let P be a parabolic subgroup such that $\sigma(P)$ is opposite to P and let N_P be the unipotent radical of P. For a compactly supported smooth function ϕ on G/H, we define $\mathcal{R}_P(\phi)(g)$ to be the integral of $N_P \ni n \mapsto \phi(gn \cdot H)$ over N_P. The Radon transform $\mathcal{R}_P$ thus obtained can be extended to a large class of distributions containing the rapidly decreasing smooth functions and the compactly supported distributions.

For these transforms we derive support theorems in which the support of ϕ is (partially) characterized in terms of the support of $\mathcal{R}_P\phi$. The proof is based on the relation between the Radon transform and the Fourier transform on G/H, and a Paley–Wiener-shift type argument. Our results generalize the support theorem of Helgason for the Radon transform on a Riemannian symmetric space.

Keywords: Radon transform; Horospherical transform; Horosphere; Support theorem; Symmetric space

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E-mail address: j.j.kuit@gmail.com.

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0. Introduction

Let G be a noncompact connected semisimple Lie group with finite center, let $G = KAN$ be an Iwasawa decomposition of G and let M be the centralizer of A in K. A horosphere in $X = G/K$ is a submanifold of the form $gN \cdot x_0$, $x_0 = e \cdot K$. The set of horospheres is isomorphic (as a G-space) to $\Xi = G/MN$ via the map $E : g \cdot MN \mapsto g \cdot \xi_0$, $\xi_0 = N \cdot x_0$. The Radon transform on X is the G-equivariant map $\mathcal{R} : C_c^\infty(X) \to C^\infty(\Xi)$ given by

$$\mathcal{R}\phi(g \cdot \xi_0) = \int_N \phi(gn \cdot x_0)\, dn.$$

In [16, Lemma 8.1] Helgason proved the following support theorem for this transform.
Let ϕ be a compactly supported smooth function on X and let V be a closed ball in X. Assume that $\mathcal{R}\phi(\xi) = 0$ whenever $E(\xi) \cap V = \emptyset$. Then $\phi(x) = 0$ for $x \notin V$.
Note that this theorem implies that $\mathcal{R}$ is injective on the space of compactly supported smooth functions.