

Presenting higher stacks as simplicial schemes

J.P. Pridham

Department of Pure Mathematics and Mathematical Statistics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WB, UK

Received 11 May 2010; accepted 30 January 2013

Available online 5 March 2013

Communicated by Bertrand Toen

Abstract

We show that an n -geometric stack may be regarded as a special kind of simplicial scheme, namely a Duskin n -hypercgroupoid in affine schemes, where surjectivity is defined in terms of covering maps, yielding Artin n -stacks, Deligne–Mumford n -stacks and n -schemes as the notion of covering varies. This formulation adapts to all HAG contexts, so in particular works for derived n -stacks (replacing rings with simplicial rings). We exploit this to describe quasi-coherent sheaves and complexes on these stacks, and to draw comparisons with Kontsevich’s dg-schemes. As an application, we show how the cotangent complex controls infinitesimal deformations of higher and derived stacks.

© 2013 Elsevier Inc. All rights reserved.

Keywords: Derived algebraic geometry; Higher stacks; Simplicial schemes

Contents

0. Introduction.....	185
1. Background on n -geometric stacks.....	188
1.1. A general setup.....	188
1.1.1. Simplicial diagrams.....	188
1.1.2. ε -morphisms.....	189
1.2. Geometric stacks.....	189
1.3. HAG contexts.....	190
1.3.1. Hyperdescent and hypersheaves.....	190
1.3.2. HA contexts and HAG contexts.....	191

E-mail address: J.P.Pridham@dpmms.cam.ac.uk.

1.3.3.	Geometric stacks	192
1.3.4.	Higher Artin and Deligne–Mumford stacks	193
1.3.5.	Derived Artin stacks	193
1.4.	Other examples	194
2.	n -hypergroupoids	196
3.	Homotopy n -hypergroupoids in model categories	201
3.1.	Definitions and basic properties	201
3.2.	Hom-spaces	203
3.3.	Morphisms via pro-objects	204
3.4.	Artin n -hypergroupoids and étale hypercoverings	207
4.	Hypergroupoids vs. n -stacks	208
4.1.	Passage to n -stacks	208
4.2.	Resolutions	209
4.3.	Morphisms	211
4.4.	From representables to affines	213
5.	Quasi-coherent modules	215
5.1.	Left Quillen presheaves and Cartesian sections	215
5.2.	Comparison with sheaves on stacks	216
5.3.	Cartesian replacement and derived direct images	217
5.4.	Sheaves on Artin n -stacks	219
5.4.1.	Quasi-coherent modules	219
5.4.2.	Quasi-coherent complexes	220
5.4.3.	Derived direct images and cohomology	221
6.	Alternative formulations of derived Artin stacks	222
6.1.	Approximation and completion	222
6.2.	Local thickenings	224
6.3.	Henselian thickenings	226
6.4.	DG stacks and dg-schemes	227
7.	Derived Quasi-coherent sheaves and the cotangent complex	231
7.1.	Cotangent complexes	232
7.1.1.	Comparison with Olsson	237
8.	Deformations of derived Artin stacks	237
8.1.	Deforming morphisms of derived stacks	237
8.2.	Deformations of derived stacks	240
	Acknowledgments	244
	References	244

0. Introduction

Although the usual approach to defining n -stacks [48,32] undoubtedly yields the correct geometric objects, it has numerous drawbacks. The inductive construction does not lend itself easily to calculations, while the level of abstract homotopy theory involved can make n -stacks seem inaccessible to many. In this paper, we introduce a far more elementary concept, namely a Duskin–Glenn n -hypergroupoid in affine schemes, and show how it is equivalent to the concept of n -geometric stacks introduced in [48]. According to [45], this is essentially the formulation of higher stacks originally envisaged by Grothendieck in [22]. It is also closely related to Zhu’s Lie n -groupoids.

In [18], Glenn defined an n -dimensional Kan hypergroupoid to be a simplicial set $X \in \mathbb{S}$ for which the horn fillers all exist, and are moreover unique in levels greater than n . Explicitly, the

Download English Version:

<https://daneshyari.com/en/article/4666075>

Download Persian Version:

<https://daneshyari.com/article/4666075>

[Daneshyari.com](https://daneshyari.com)