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Advances in Mathematics 238 (2013) 184–245

ADVANCES IN
Mathematics

www.elsevier.com/locate/aim

Presenting higher stacks as simplicial schemes

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Received 11 May 2010; accepted 30 January 2013
Available online 5 March 2013

Communicated by Bertrand Toen

Abstract

We show that an n-geometric stack may be regarded as a special kind of simplicial scheme, namely a Duskin n-hypergroupoid in affine schemes, where surjectivity is defined in terms of covering maps, yielding Artin n-stacks, Deligne–Mumford n-stacks and n-schemes as the notion of covering varies. This formulation adapts to all HAG contexts, so in particular works for derived n-stacks (replacing rings with simplicial rings). We exploit this to describe quasi-coherent sheaves and complexes on these stacks, and to draw comparisons with Kontsevich's dg-schemes. As an application, we show how the cotangent complex controls infinitesimal deformations of higher and derived stacks.
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Keywords: Derived algebraic geometry; Higher stacks; Simplicial schemes

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doi:10.1016/j.aim.2013.01.009

0. Introduction

Although the usual approach to defining n-stacks [48,32] undoubtedly yields the correct geometric objects, it has numerous drawbacks. The inductive construction does not lend itself easily to calculations, while the level of abstract homotopy theory involved can make n-stacks seem inaccessible to many. In this paper, we introduce a far more elementary concept, namely a Duskin–Glenn n-hypergroupoid in affine schemes, and show how it is equivalent to the concept of n-geometric stacks introduced in [48]. According to [45], this is essentially the formulation of higher stacks originally envisaged by Grothendieck in [22]. It is also closely related to Zhu's Lie n-groupoids.

In [18], Glenn defined an n-dimensional Kan hypergroupoid to be a simplicial set $X \in \mathbb{S}$ for which the horn fillers all exist, and are moreover unique in levels greater than n. Explicitly, the

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