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Generic bifurcation of refracted systems

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Abstract

In this article we discuss some qualitative and geometric aspects of non-smooth dynamical systems theory. Our goal is to study the diagram bifurcation of typical singularities that occur generically in one parameter families of certain piecewise smooth vector fields named *Refracted Systems.* Such systems has a codimension-one submanifold as its discontinuity set.

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1. Introduction

It is fairly known that a large number of problems from mechanics, electrical engineering and the theory of automatic control are described by non-smooth systems; see [1]. Some basic methods of the qualitative theory are established and developed in [8], and in a large number of papers [3–5,9,12].

In this paper we study (germs of) piecewise-smooth system on $\mathbb{R}^3, 0$ and $\Sigma \subset \mathbb{R}^3$ be given by $\Sigma = f^{-1}(0)$, where f is (a germ of) a smooth function $f : \mathbb{R}^3, 0 \longrightarrow \mathbb{R}$, $f(0) = 0$, having $0 \in \mathbb{R}$ as a regular value (i.e. $\nabla f(p) \neq 0$, for any $p \in f^{-1}(0)$).

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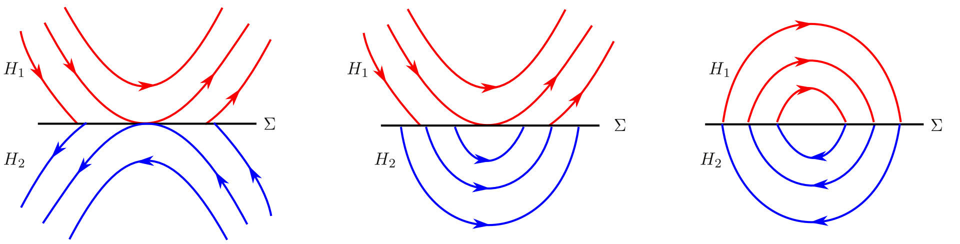

Fig. 1. Generic typical singularities of refracted Hamiltonian vector fields.

Clearly Σ is the separating boundary of the regions $\Sigma_+ = \{q \in \mathbb{R}^3, 0 | f(q) \geq 0\}$ and $\Sigma_- = \{q \in \mathbb{R}^3, 0 | f(q) \leq 0\}$. We can assume without the loss of generality that Σ is represented, locally around a point $q = (x, y, z)$, by the function $f(x, y, z) = z$.

Designate by χ^r the space of C^r vector fields on $\mathbb{R}^3, 0$ endowed with the C^r-topology with $r \geqslant 1$ or $r = \infty$, big enough for our purposes. Call Ω^r the space of vector fields $Z : \mathbb{R}^3, 0 \longrightarrow \mathbb{R}^3$ such that

$$Z(x, y, z) = \begin{cases} X(x, y, z), & \text{for } (x, y, z) \in \Sigma_+, \\ Y(x, y, z), & \text{for } (x, y, z) \in \Sigma_-, \end{cases} \tag{1}$$

where $X = (X_1, X_2, X_3), Y = (Y_1, Y_2, Y_3) \in \chi^r$. We write $Z = (X, Y) \in \chi^r \times \chi^r = \Omega^r$. Endow Ω^r with the product topology.

The trajectories of Z are solutions of the autonomous differential equation system $\dot{q} = Z(q)$, which has, in general, discontinuous right-hand side. See [8] for basic concepts and results of ordinary differential equations with discontinuous right-hand side. Related topics can be found in [11,14,17].

In what follows we use the notation

$$Xf(p) = \langle \nabla f(p), X(p) \rangle .$$

We are interested in the study of discontinuous systems having the property $Xf(p) = Yf(p)$ for all $p \in \Sigma$. These systems are known as *refracted systems*.

Our main motivation to study refracted systems comes from the remarkable work of I. Ekeland in [7] where the main problem in the classical calculus of variations was carried out by means of the classification in $2D$ of generic typical singularities of refracted Hamiltonian vector fields. Their dynamics are illustrated in Fig. 1.

It is worth mentioning that in [2] a class of refracted systems in R^n, known as relay systems, is discussed. They have the form:

$$X = A\,x + \operatorname{sgn}(x_1)\,k$$

where $x = (x_1, x_2, \ldots, x_n)$, $A \in M_R(n, n)$ and $k = (k_1, k_2, \ldots, k_n)$ is a constant vector in R^n.

In [10] the generic singularities of relay systems in $4D$ were studied. In this paper, conditions for a version of the Lyapunov Center Theorem were obtained.

As said before, throughout the text we assume that $f(x, y, z) = z$. So $\Sigma = \{(x, y, 0)\}$ and the condition $Xf(p) = Yf(p)$ for all $p \in \Sigma$ is equivalent to $X^3(x, y, 0) = Y^3(x, y, 0)$. Call Ω^r_{Ref} the set of all refracted systems in Ω^r.