

Combinatorics and topology of straightening maps, I: Compactness and bijectivity

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Abstract

We study the parameter space structure of degree $d \geq 3$ one complex variable polynomials as dynamical systems acting on $\mathbb{C}$. We introduce and study *straightening maps*. These maps are a natural higher degree generalization of the ones introduced by Douady and Hubbard to prove the existence of small copies of the Mandelbrot set inside itself. We establish that straightening maps are always injective and that their image contains all the corresponding hyperbolic systems. Also, we characterize straightening maps with compact domains. Moreover, we give two classes of bijective straightening maps. The first produces an infinite collection of embedded copies of the $(d - 1)$ -fold product of the Mandelbrot set in the connectedness locus of degree $d \geq 3$. The second produces an infinite collection of full families of quadratic connected filled Julia sets in the cubic connectedness locus, such that each filled Julia set is quasiconformally embedded.

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Keywords: Complex dynamics; Renormalizations; Straightening maps; Connectedness loci

Contents

0. Introduction.....	2668
1. Outline of results.....	2670
1.1. Combinatorially renormalizable polynomials.....	2670

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1.2.	Mapping schemata.....	2670
1.3.	The universal polynomial model space.....	2671
1.4.	Polynomial-like maps over schemata.....	2671
1.5.	External markings.....	2672
1.6.	Standard external marking of polynomial maps in $\text{Poly}(T)$	2673
1.7.	Straightening of polynomial-like maps over reduced schemata.....	2673
1.8.	Renormalizable polynomials.....	2674
1.9.	Internal angle systems and induced external markings.....	2675
1.10.	Straightening maps and injectivity.....	2676
1.11.	Onto hyperbolic maps.....	2676
1.12.	Compactness.....	2678
1.13.	Cubic reduced mapping schemata.....	2678
1.14.	Capture and disjoint type straightenings.....	2679
2.	Polynomial-like maps.....	2680
2.1.	Proof of the straightening theorem.....	2680
3.	Laminations.....	2682
3.1.	Laminations.....	2682
3.2.	Invariant rational laminations: unlinked classes, fibers and critical elements.....	2682
3.3.	Renormalizations.....	2687
3.4.	Proof of Proposition 3.7.....	2689
4.	Yoccoz puzzles.....	2690
4.1.	A combinatorial Yoccoz puzzle.....	2690
4.2.	A Yoccoz puzzle.....	2691
4.3.	From combinatorial puzzles to laminations.....	2694
5.	Combinatorial surjectivity of straightening.....	2696
5.1.	Combinatorial straightening.....	2697
5.2.	Combinatorial tuning.....	2698
5.3.	Combinatorial straightening and combinatorial tuning.....	2700
5.4.	The renormalizable set.....	2701
5.5.	Combinatorial surjectivity.....	2707
6.	Injectivity of straightening maps.....	2708
6.1.	Proof of Theorem B.....	2709
6.2.	Proof of the Key Lemma.....	2712
7.	Onto hyperbolic maps.....	2717
7.1.	Onto hyperbolic maps.....	2717
7.2.	Proof of Theorem C.....	2719
8.	Compactness.....	2720
9.	Surjectivity of straightening maps.....	2725
9.1.	Preliminaries.....	2725
9.2.	Surjectivity of straightening: primitive disjoint type.....	2726
9.3.	Surjectivity of straightening: cubic primitive capture type.....	2727
9.4.	Complex submanifolds and quasiconformality of straightening.....	2729
9.4.1.	Proof of Theorem E.....	2729
9.4.2.	Capture case.....	2730
	Acknowledgment.....	2732
	References.....	2732

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