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Geometry and analysis of Dirichlet forms

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Abstract

Let $\mathscr{E}$ be a regular, strongly local Dirichlet form on $L^2(X, m)$ and d the associated intrinsic distance. Assume that the topology induced by d coincides with the original topology on X, and that X is compact, satisfies a doubling property and supports a weak $(1, 2)$-Poincaré inequality. We first discuss the (non-)coincidence of the intrinsic length structure and the gradient structure. Under the further assumption that the Ricci curvature of X is bounded from below in the sense of Lott–Sturm–Villani, the following are shown to be equivalent:

(i) the heat flow of $\mathscr{E}$ gives the unique gradient flow of $\mathscr{U}_\infty$,
(ii) $\mathscr{E}$ satisfies the Newtonian property,
(iii) the intrinsic length structure coincides with the gradient structure.

Moreover, for the standard (resistance) Dirichlet form on the Sierpinski gasket equipped with the Kusuoka measure, we identify the intrinsic length structure with the measurable Riemannian and the gradient structures. We also apply the above results to the (coarse) Ricci curvatures and asymptotics of the gradient of the heat kernel.
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Keywords: Dirichlet form; Intrinsic distance; Length structure; Differential structure; Sierpinski gasket; Gradient flow; Ricci curvature; Poincaré inequality; Metric measure space

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Contents

1. Introduction

It is well known that on $\mathbb{R}^n$, associated to the Dirichlet energy

$$\int_{\mathbb{R}^n} |\nabla f(x)|^2 dx,$$

there is a naturally defined heat semigroup (flow). Jordan et al. [17] and Otto [35] understood this heat flow as a gradient flow of the Boltzmann–Shannon entropy with respect to the L^2-Wasserstein metric on the space of probability measures on $\mathbb{R}^n$. Since then this has been extended to Riemannian manifolds, Finsler manifolds, Heisenberg groups, Alexandrov spaces and metric measure spaces; see, for example, [35,1,51,9,18,33,13,2]. The gradient flow has also attracted considerable attention in various settings; see, for example, [1,13,51,12] and the reference therein. In particular, the works [1,12,13] in abstract setting motivate one to extend the above phenomenon of [17] to settings such as metric measure spaces with Ricci curvatures of Lott et al. [49,50,29] bounded from below.

Moreover, a heat semigroup (flow) is naturally associated to any given Dirichlet form. Via this, a notion of Ricci curvature bounded from below was introduced by Bakry and Emery [4]. Observe that the Ricci curvature of Bakry–Emery essentially depends on the differential (gradient) structure. On the other hand, under some additional assumptions on the underlying metric measure space, a notion of Ricci curvature bounded from below was introduced by Lott et al. [29,49,50], purely in terms of the length structure. It is then natural to analyze the connections between these different approaches; see [13,2] for seminal studies in this direction. In this paper, we consider the intrinsic length structures and gradient structures of Dirichlet forms.

Let X be a locally compact, connected and separable Hausdorff space and m a nonnegative Radon measure with support X. Let $\mathscr{E}$ be a regular, strongly local Dirichlet form on $L^2(X)$, Γ the squared gradient and d the intrinsic distance induced by $\mathscr{E}$. We always assume that the topology induced by d coincides with the original topology on X.

In Section 2, we establish the coincidence of the intrinsic length structure and the gradient structure of Dirichlet forms under a doubling property, a weak Poincaré inequality and the Newtonian property. Indeed, we prove that if (X,d,m) satisfies the doubling property, then for every $u \in \mathrm{Lip}(X)$, the energy measure $\Gamma(u,u)$ is absolutely continuous with respect to m and $\frac{d}{dm}\Gamma(u,u) \le (\mathrm{Lip}\, u)^2$ almost everywhere; see Theorem 2.1. If we further assume that (X,d,m) supports a weak $(1,p)$-Poincaré inequality for some $p \in [1,\infty)$ and that $(X,\mathscr{E},m)$ satisfies the Newtonian property introduced in this paper, then $\frac{d}{dm}\Gamma(u,u) = (\mathrm{Lip}\, u)^2$ almost everywhere; see Theorem 2.2.

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