The Fibonacci partition triangles

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Abstract

In two previous papers we have presented partition formulas for the Fibonacci numbers motivated by the appearance of the Fibonacci numbers in the representation theory of the 3-Kronecker quiver and its universal cover, the 3-regular tree. Here we show that the basic information can be rearranged in two triangles. They are quite similar to the Pascal triangle of the binomial coefficients, but in contrast to the additivity rule for the Pascal triangle, we now deal with additivity along "hooks", or, equivalently, with additive functions for valued translation quivers. As for the Pascal triangle, we see that the numbers in these Fibonacci partition triangles are given by evaluating polynomials. We show that the two triangles can be obtained from each other by looking at differences of numbers, it is sufficient to take differences along arrows and knight's moves.

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The aim of the paper is to rearrange the positive integers which are used in the partition formulas for the Fibonacci numbers as considered in [3,4]. For the even-index Fibonacci numbers we obtain a proper triangle which we call the even-index Fibonacci partition triangle. Second, what we call the odd-index Fibonacci partition triangle actually is a triangle only after removing one number (but it seems worthwhile to take this additional position into account). These arrangements of integers are quite similar to the Pascal triangle of the binomial coefficients. In particular, we will show that the numbers along the inclined lines are given by evaluating polynomials.
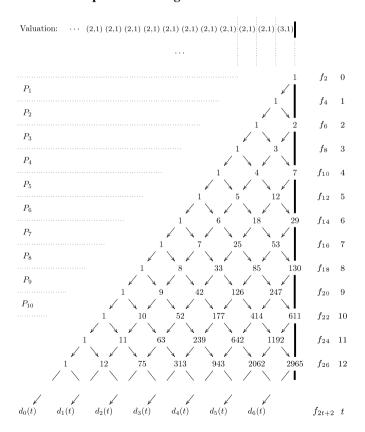
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Let us recall that our work on the partition formulas was based on the appearance of the Fibonacci numbers in the representation theory of quivers and the present paper again relies on concepts which have been developed in this context. Namely, both triangles turn out to show additive functions on valued translation quivers. Translation quivers have been considered frequently in the representation theory of quivers and finite-dimensional algebras. The valued translation quivers can be used in order to describe module categories, but those with non-trivial valuation (as in the case of the Fibonacci partition triangles) have seldom be seen to be of importance when dealing with quivers.

In Sections 1 and 2 we will exhibit the even-index, and the odd-index triangle, respectively. Our main task will be to show in which way the two triangles can be obtained from each other, see Sections 3 and 4. The relationship which we will encounter shows that the two triangles are intimately connected. The proof provided here relies on the categorification of the Fibonacci pairs given in [4], it will be given in Section 5. The final Section 6 provides some further remarks and open questions. For known properties of the Fibonacci numbers one may consult the book of Koshy [7], see also [9].

The investigation is based on the Fibonacci partition formulas established in [3] for the Fibonacci numbers with even index, and in the Ph.D. Thesis [2] of Fahr for those with odd index, see also [4], but also on further discussions of the authors during the time when Fahr was a Ph.D. student at Bielefeld. The final version was written by Ringel.

1. The even-index Fibonacci partition triangle

Valuation: $\cdots$ (2,1) (2,1) (2,1) (2,1) (2,1) (2,1) (2,1) (2,1) (2,1) (3,1)

$\cdots$

P_1						1	f_2 0
P_2						1	f_4 1
P_3					1	2	f_6 2
P_4				1	3		f_8 3
P_5			1	4	7		f_{10} 4
P_6		1	5	12			f_{12} 5
P_7	1	6	18	29			f_{14} 6
P_8	1	7	25	53			f_{16} 7
P_9	1	8	33	85	130		f_{18} 8
P_{10}	1	9	42	126	247		f_{20} 9
	1	10	52	177	414	611	f_{22} 10
	1	11	63	239	642	1192	f_{24} 11
	1	12	75	313	943	2062	2965	f_{26} 12

$d_0(t)$ $d_1(t)$ $d_2(t)$ $d_3(t)$ $d_4(t)$ $d_5(t)$ $d_6(t)$ f_{2t+2} t