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On the existence of an MVU estimator for target localization with censored, noise-free binary detectors[☆]

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ABSTRACT

The problem of target localization with censored, noise-free binary detectors is considered. In this setting only the detecting sensors report their locations to the fusion center. It is proven that if the radius of detection is unknown to the fusion center, a minimum variance unbiased (MVU) estimator does not exist. Also it is shown that when the radius is known the center of mass of the possible target region is the MVU estimator among estimators that are invariant under Euclidean motion. In addition, a sub-optimum estimator is introduced whose performance is close to the MVU estimator but is preferred computationally. Moreover, for the case when the radius of detection is unknown a sub-optimum estimator is proposed that performs close to the Clairvoyant estimator. Furthermore, minimal sufficient statistics have been provided, both when the detection radius is known and when it is not. Simulations confirmed that the derived MVU estimator outperforms several heuristic location estimators.

1. Introduction

Localization of an unknown transmitter with observations from a network of sensors is a well known problem in the literature [1,2]. The observations can be carried out through measurement of Angle of Arrival (AoA) [3–5], Time Difference of Arrival (TDoA) [6–8], or Received Signal Strength (RSS) [9–23]. When the sensors are mobile, Frequency Difference of Arrival (FDoA) can also be used as an additional source of information [24–26]. AoA, TDoA and FDoA approaches require sophisticated sensors, and, therefore do not fit well within the energy and complexity limitations of wireless sensor networks. In practice, an exact(un-quantized) RSS measurement is unrealistic because it requires unlimited bandwidth to communicate the data to the fusion center. A binary RSS measurement is preferred because it is simpler and requires less resources [18–23]. In some papers it is assumed that the propagation model is isotropic and detection is noise-free [20–23]. For a transmitter which is not bursty, noise in each sensor can be eliminated effectively by averaging the power measurements over a long period of time [23]. Moreover, the noise-free regime provides a lower bound for location estimation in the presence of uncertainty such as noise or fading. Therefore, a minimum variance unbiased (MVU) estimator of this scenario will serve as a benchmark for comparison of the performance of all target localizers. In addition, it provides insight into the effect of parameters

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Nomenclature

$\mathbb{G}$	Region of sensor deployment
$\mathcal{A}(\mathbb{G})$	Hyper volume (area) of the region $\mathbb{G}$
$\mathbf{z}_\mathrm{T}$	Target location in the space
$\mathbf{O}$	Origin
N	Total number of sensors deployed in the region
n	Number of detecting sensors
$\mathcal{S}$	Set of indices of all detecting sensors
$\mathcal{Z}$	Set of locations of the detecting sensors
$\mathbf{Z}$	Matrix of locations of the detecting sensors
$\mathbf{Z}_k$	Matrix of locations of the first k detecting sensors
$\mathcal{B}_R(\mathbf{z}_i)$	A ball with radius R around the i'th sensor
$\mathcal{T}(\mathbf{Z})$	The *possible target region* based on $\mathbf{Z}$ observation
$\mathcal{T}_j(\mathbf{Z})$	The *possible target region* with sensor jth excluded from evaluation
$\mathcal{S}_\mathrm{MSS}$	Indices of the sensors forming the minimal sufficient statistic
$\mathcal{Z}_\mathrm{MSS}$	Set of locations of the sensors forming the minimal sufficient statistic
$\mathbf{Z}_\mathrm{MSS}$	Matrix of locations of the sensors forming the minimal sufficient statistic
$\mathcal{MSS}(\mathbf{Z})$	The function generating $\mathcal{Z}_\mathrm{MSS}$ from $\mathbf{Z}$
$MSS(\mathbf{Z})$	The function generating $\mathbf{Z}_\mathrm{MSS}$ from $\mathbf{Z}$
$\mathbf{1}_X$	The indicator function of X
$\sigma(\mathcal{T})$	Boundary surface of $\mathcal{T}$
$f(\mathbf{Z};\mathbf{z}_\mathrm{T})$	Probability density function of occurrence of $\mathbf{Z}$ if the target located at $\mathbf{z}_\mathrm{T}$
$\mathcal{R}_\Theta(\mathbf{X})$	Range of random variable $\mathbf{X}$ over parameter space Θ
$\mathcal{C}(\mathcal{Z})$	Convex hull of the set, $\mathcal{Z}$
$\mathcal{N}(\mathcal{T})$	Set of points whose maximum distance from $\mathcal{T}$ are not more than R
$\mathbf{T_k}$	A matrix storing the shift of $(\mathbf{z}_1,\ldots,\mathbf{z}_k)$ elements from $\mathbf{z}_1$
$g(\mathbf{Z})$	An estimator based on observation $\mathbf{Z}$
$CM(\mathcal{T}(\mathbf{Z},R_1))$	Center of mass of $\mathcal{T}(\mathbf{Z})$ with radius R_1

such as density of sensor deployment and power of transmitter in the localization process. This assumption reduces the detection problem to the question whether the target is located within a certain radius of each sensor or not. However, as we will see in Section 4 the estimation problem is not well behaved, and Cramér–Rao bound (CRB) cannot be established. In this paper, we consider the problem in a censored scenario where the target location is estimated based on the detecting sensors' data. Similar approach is used in [20,27]. The motivation is to save on communication and processing load related to non-detecting sensors when the region of sensor deployment is much larger than the detection radius [27].

The rest of the paper is organized as follows: Section 2 is the problem formulation. In Section 3, we study the redundancy in information provided by the detecting sensors and find out minimal sufficient statistics for them. The existence of an MVU estimator is investigated in Section 4. In Section 5 some sub-optimal estimators with low computational complexity that perform close to optimal are presented. The performance of the proposed methods are compared with some heuristic ones through simulation in Section 6. Concluding remarks and future expansions are discussed in Section 7 and Section 8 respectively.

2. Problem formulation

Assume that a target is located at an unknown location $\mathbf{z}_\mathrm{T} = [x_{1_\mathrm{T}},\ldots,x_{l_\mathrm{T}}]$ in l-dimensional space (in practical applications l is either 2 or 3) and transmits a signal whose power propagates isotropically and is attenuated monotonically as a function of distance from the target. N sensors, are randomly scattered in a deployment region $\mathbb{G}$, with hyper volume $\mathcal{A}(\mathbb{G})$. They measure the received power and compare it with a threshold, τ, to make a binary decision about the target presence. We assume the sensors make a noise-free decision, which can be considered as the limiting case when the measured power is averaged over a sufficiently long duration. Furthermore, the sensors are configured such that only the detecting sensors report their locations, $\mathbf{z}_1,\ldots,\mathbf{z}_n$, to the fusion center where the localization decision is performed. Since the received power is a decreasing function of distance from the target, there is a ball around the target, $\mathcal{B}_R(\mathbf{z}_\mathrm{T})$, where all the sensors inside will detect the target and those outside will not. From now on we call R the detection radius. We assume that $\mathbb{G}$ is sufficiently large such that $\mathcal{B}_{2R}(\mathbf{z}_\mathrm{T}) \subset \mathbb{G}$. In addition, we assume that at least one sensor detects the target. Let n be the number of detecting sensors ($n \geq 1$) and $\mathcal{S} = \{1,\ldots,n\}$ be the set of indices of all detecting sensors. Therefore, $\mathcal{Z} = \{\mathbf{z}_i | i \in \mathcal{S}\}$ will be the set of locations of all detecting sensors and $\mathbf{Z} = (\mathbf{z}_i | i \in \mathcal{S})$ denotes the matrix containing those locations.

To solve the problem in each stage, we have divided the problem into two separate cases: (I) when the radius of detection is known which is equivalent to the situation when propagation model and the transmit power are known to the fusion center; and (II) when the radius of detection is unknown which is equivalent to the case when propagation model or transmit power are unknown such as the case in non-cooperative localization.

3. Sufficient statistics

In the this section we derive sufficient statistics for estimation of target location $\mathbf{z}_\mathrm{T}$. We consider the problem in two cases depending on whether the detection radius, R, is known or unknown.

3.1. Known detection radius

Let us define the *possible target region* given observation $\mathbf{Z}$ as [28,29]

$$\mathcal{T}(\mathbf{Z}) = \bigcap_{i\in\mathcal{S}} \mathcal{B}_R(\mathbf{z}_i). \tag{1}$$

Alternatively since the order of $\mathbf{Z}$ elements does not matter in this definition, we may equivalently define $\mathcal{T}$ over the $\mathcal{Z}$