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Total graph of a module with respect to singular submodule

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Abstract. Let R be a commutative ring with unity and M be an R-module. We introduce the total graph of a module M with respect to singular submodule $Z(M)$ of M as an undirected graph $T(\Gamma(M))$ with vertex set as M and any two distinct vertices x and y are adjacent if and only if $x + y \in Z(M)$. We investigate some properties of the total graph $T(\Gamma(M))$ and its induced subgraphs $Z(\Gamma(M))$ and $\overline{Z}(\Gamma(M))$. In some aspects, we have noticed some sort of finiteness.

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1. Introduction

In 1988, Istvan Beck [10] opened up the fascinating insight which relates a graph with the algebraic structure ring. He introduced the zero divisor graph of a commutative ring, and later on, this introduction was slightly modified by D.D. Anderson and M. Naseer in [7]. Further modification to the concept of the zero-divisor graph was made in [6]. Many authors studied the zero-divisor graph in the sense of Anderson–Livingston as in [6]. Since then, the concept of the zero divisor graph of ring has been playing a vital rule in its expansion. Motivating from this well expanded idea of Beck, lots of correspondences of a graph with algebraic structures have been introduced with a variety of applications. Some of them are

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the comaximal graph of a commutative ring by Sharma and Bhatwadekar [16], the total graph of commutative ring by Anderson and Badawi [4], the intersection graph of ideals of a ring by Chakrabarty et al. [11], etc.

In 2008, Anderson and Badawi [4] defined the total graph of a commutative ring R, which is an undirected graph with vertex set as R with any two vertices are adjacent if and only if its ring sum is a zero divisor of R. In that paper, they discussed the characteristics of total graph and its two induced subgraphs by considering two cases, namely, the set of zero divisors $Z(R)$ of R is an ideal of R and $Z(R)$ is not an ideal of R. Thereafter, Akbari et al. [3] continued this concept of total graph of commutative rings. Ahmad Abbasi and Shokoofe Habibi [1] discussed the total graph of a commutative ring with respect to the proper ideals. Anderson and Badawi [5] interpreted the total graph of a commutative ring without zero element. In [17], M.H. Shekarriz et al. observed some basic graph theoretic properties of the total graph of a finite commutative ring. The prospect for total graph of modules is also observed in recent times. A. Abbasi and S. Habibi [2] investigated the total graph of a commutative ring with respect to the proper submodules of a module. The total torsion element graph of a module over a commutative ring was introduced by S. Atani and S. Habibi [8]. The above module based concepts of total graph extend the work of Anderson and Badawi [4].

In this article, we introduce the notion of singularity of a module over a ring and define the total graph of a module M with respect to singular submodule $Z(M)$. Before going to our discussion we recall the following.

Let R be a commutative ring. An element x of R is called a zero-divisor of R if there exists a non-zero element y of R with $xy = 0$. The collection of all zero-divisors of R is denoted by $Z(R)$, and henceforth, we use it. An ideal I of R is an essential ideal if its intersection with any non-zero ideal of R is non-zero. For the R-modules M and N, a mapping $f : M \to N$ is said to be a module homomorphism if $f(x + y) = f(x) + f(y)$ and $f(rx) = rf(x)$ for all $x, y \in M$ and $r \in R$. If f is also one-one, then it is said to be a module monomorphism. A one-one and onto module homomorphism is called a module isomorphism.

Throughout this discussion, all graphs are undirected. Let G be an undirected graph with the vertex set $V(G)$, unless otherwise mentioned. If G contains n vertices then we write $|V(G)| = n$. Two graphs G and H are isomorphic if there exists a one-to-one correspondence between their vertex sets which preserves adjacency. A subgraph of G is a graph having all of its vertices and edges in G. A spanning subgraph of G contains all vertices of it. For any set S of vertices of G, the induced subgraph $\langle S\rangle$ is the maximal subgraph of G with vertex set S. Thus two points of S are adjacent in $\langle S\rangle$ if and only if they are adjacent in G. The degree of a vertex v in a graph G is the number of edges incident with v. The degree of a vertex v is denoted by $deg(v)$. The vertex v is $isolated$ if $deg(v) = 0$. A walk in G is an alternating sequence of vertices and edges, $v_0x_1v_1...x_nv_n$ in which each edge x_i is $v_{i-1}v_i$. The length of such a walk is n, the number of occurrences of edge in it. A closed walk has the same first and last vertices. A path is a walk in which all vertices are distinct; a cycle or circuit is a closed walk with all points distinct (except the first and last). A cycle of length 3 is called a triangle. An acyclic graph does not contain a cycle. G is connected if there is a path between every two distinct vertices. A graph which is not connected is called a disconnected graph. A totally disconnected graph does not contain any edges. For distinct vertices x and y of G, let $d(x, y)$ be the length of the shortest path from x to y and if there is no such path we define $d(x, y) = \infty$. The eccentricity $e(v)$ of a vertex v in a connected graph G is $max\ d(u, v)$ for all u in $V\ (G)$. A vertex with minimum eccentricity is called a center